

GCMP

**GREATER
CLEVELAND
MATHEMATICS
PROGRAM**



INTERMEDIATE UNIT

3

S R A

REORDER NO. 3-69043

3

Intermediate Unit

GCMP

GREATER CLEVELAND MATHEMATICS PROGRAM

Prepared by the staff of the
Educational Research Council of
Greater Cleveland, under the
direction of George S. Cunningham

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S R A

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11 THE DIVISION ALGORITHM

We can use the equation $b = aq + r$ to find the greatest quotient (q) and least remainder (r).

$$\begin{aligned}58 &= 8q + r \\58 &= 8 \times 3 + 34 \\34 &= 8 \times 4 + 2 \\58 &= (8 \times 3) + (8 \times 4) + 2 \\&= 8 \times (3 + 4) + 2 \\&= 8 \times 7 + 2\end{aligned}$$

Find the greatest quotient (q) and least remainder (r).

1. $76 = 9q + r$

2. $119 = 7q + r$

3. $97 = 8q + r$

4. $66 = 9q + r$

5. $43 = 7q + r$

6. $113 = 8q + r$

7. $217 = 6q + r$

8. $186 = 7q + r$

9. $347 = 11q + r$

10. $396 = 12q + r$

11. $496 = 19q + r$

12. $1312 = 4q + r$

13. $1822 = 11q + r$

14. $1966 = 25q + r$

15. $200 = 37q + r$

16. $1000 = 99q + r$

17. $516 = 13q + r$

18. $777 = 4q + r$

Let's see how we can use the division algorithm to find the greatest quotient and the least remainder.

$$226 = 7q + r$$

$$226 = 7 \times 5 + 191$$

Record the product of 7 and 5. \longrightarrow

Record the remainder. \longrightarrow

$$\begin{array}{r} 7 \\ \hline 226 \\ 35 \\ \hline 191 \\ 191 \\ \hline 5 \end{array}$$

$$191 = 7 \times 20 + ?$$

Record the product of 7 and 20. \longrightarrow

Record the remainder. \longrightarrow

$$\begin{array}{r} 7 \\ \hline 191 \\ 140 \\ \hline 51 \\ 51 \\ \hline 20 \end{array}$$

$$51 = 7 \times 7 + ?$$

Record the product of 7 and 7. \longrightarrow

Record the remainder. \longrightarrow

$$\begin{array}{r} 7 \\ \hline 51 \\ 49 \\ \hline 2 \\ 2 \\ \hline 7 \end{array}$$

Therefore:

$$\begin{aligned} 226 &= 7 \times (5 + 20 + 7) + 2 \\ &= 7 \times 32 + 2 \end{aligned}$$

Now combine the steps you used to compute the greatest quotient.

$$\begin{array}{r} 7 \\ \hline 226 \\ ? \\ \hline ? \\ \hline ? \end{array}$$

5 \longleftarrow $226 = 7 \times 5 + ?$

20 \longleftarrow $? = 7 \times 20 + ?$

7 \longleftarrow $? = 7 \times 7 + ?$

The result of our computation is $226 = 7 \times ? + ?$.

Complete the example to find the greatest quotient and least remainder. Complete the multiplication equation to check your results.

$$409 = 12q + r$$

12 $\overline{)409}$? $\overline{\quad}$? $\overline{\quad}$? $\overline{\quad}$? $\overline{\quad}$	10 ← $409 = 12 \times 10 + \underline{\quad} ?$ 10 ← $\underline{\quad} ? = 12 \times 10 + \underline{\quad} ?$ 10 ← $\underline{\quad} ? = 12 \times 10 + \underline{\quad} ?$ 4 ← $\underline{\quad} ? = 12 \times 4 + \underline{\quad} ?$
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$$\text{Therefore, } 409 = 12 \times \underline{\quad} + \underline{\quad}.$$

Compute. Write an equation to check your results.

1. $5 \overline{)729}$

2. $6 \overline{)437}$

3. $3 \overline{)191}$

4. $8 \overline{)431}$

5. $12 \overline{)329}$

6. $9 \overline{)735}$

7. $5 \overline{)427}$

8. $3 \overline{)224}$

9. $14 \overline{)725}$

10. $4 \overline{)371}$

11. $7 \overline{)656}$

12. $5 \overline{)789}$

13. $9 \overline{)342}$

14. $6 \overline{)519}$

15. $9 \overline{)347}$

16. $13 \overline{)334}$

17. $4 \overline{)295}$

18. $6 \overline{)110}$

19. $3 \overline{)397}$

20. $5 \overline{)712}$

Find the greatest quotient and the least remainder for each exercise.

$$1. \quad 4 \overline{)365}$$

$$2. \quad 9 \overline{)813}$$

$$3. \quad 3 \overline{)339}$$

$$4. \quad 7 \overline{)284}$$

$$5. \quad 5 \overline{)252}$$

$$6. \quad 4 \overline{)163}$$

$$7. \quad 9 \overline{)631}$$

$$8. \quad 6 \overline{)547}$$

$$9. \quad 8 \overline{)723}$$

$$10. \quad 12 \overline{)481}$$

$$11. \quad 11 \overline{)776}$$

$$12. \quad 23 \overline{)693}$$

$$13. \quad 14 \overline{)287}$$

$$14. \quad 50 \overline{)306}$$

$$15. \quad 24 \overline{)483}$$

$$16. \quad 16 \overline{)327}$$

$$17. \quad 19 \overline{)389}$$

$$18. \quad 15 \overline{)3012}$$

$$19. \quad 21 \overline{)543}$$

$$20. \quad 14 \overline{)735}$$

$$21. \quad 19 \overline{)610}$$

$$22. \quad 27 \overline{)1029}$$

$$23. \quad 13 \overline{)536}$$

$$24. \quad 28 \overline{)870}$$

A product of a whole number and 10 is a *multiple* of 10. For example, 50 is ? \times 10, or ? tens. Therefore 50 is a multiple of 10.

Complete each sentence.

1. $7 \times 10 = \underline{\quad}$ tens = 70

2. $3 \times 10 = \underline{\quad} \underline{\quad} = 30$

3. $12 \times 10 = \underline{\quad} \underline{\quad} = \underline{\quad}$

4. $14 \times 10 = \underline{\quad} \underline{\quad} = \underline{\quad}$

5. $60 \times 10 = \underline{\quad} \underline{\quad} = \underline{\quad}$

6. $4 \times 10 = \underline{\quad} \underline{\quad} = \underline{\quad}$

7. $25 \times 10 = \underline{\quad}$ tens = ?

8. $10 \times 10 = \underline{\quad} \underline{\quad} = \underline{\quad}$

Multiples of 100 and 1000 are also multiples of ten.

9. $1500 = \underline{\quad}$ hundreds = ? tens.

10. $3000 = \underline{\quad}$ thousands = ? hundreds = 300 ?.

Complete each sentence.

11. 240 is ? tens.

12. 3010 is ? tens.

13. 1500 is ? hundreds, or 150 ?.

14. 700 is 7 ?, or ? tens.

15. 2500 is 25 ?, or ? tens.

16. 570 is ? tens.

17. ? \times 10 = 100

18. $1 \times \underline{\quad} = 100$

19. $10 \div \underline{\quad} = 1$

20. $10 \times \underline{\quad} = 1000$

21. ? \times 10 = 10

22. ? = $1000 \div 100$

Copy and complete the table.

	Factor	Factor	Product
23.	?	hundreds	hundreds
24.	tens	?	hundreds
25.	?	tens	thousands
26.	tens	?	tens
27.	ones	?	hundreds
28.	?	hundreds	thousands
29.	?	ones	tens

Copy and complete the table.

\times	8	80	800	8000
1. 3	?	?	?	?
2. 5	?	?	?	?
3. 7	?	?	?	?
4. 9	?	?	?	?
5. 8	?	?	?	?
6. 6	?	?	?	?
7. 4	?	?	?	?
8. 2	?	?	?	?

Complete.

$$\begin{array}{l} 9. \quad 8 \times ? = 56 \\ \underline{?} \times 8 = 560 \\ 80 \times ? = 560 \\ \underline{?} \times 80 = 5600 \end{array}$$

$$\begin{array}{l} 10. \quad 9 \times ? = 72 \\ \underline{?} \times 9 = 720 \\ 90 \times ? = 720 \\ \underline{?} \times 90 = 7200 \end{array}$$

$$\begin{array}{l} 11. \quad 7 \times ? = 63 \\ 7 \times ? = 630 \\ \underline{?} \times 70 = 630 \\ 70 \times ? = 6300 \end{array}$$

$$\begin{array}{l} 12. \quad 48 = ? \times 8 \\ 480 = 8 \times ? \\ 480 = ? \times 80 \\ 4800 = ? \times 80 \end{array}$$

$$\begin{array}{l} 13. \quad 54 = 6 \times ? \\ 540 = 6 \times ? \\ 540 = ? \times 60 \\ 5400 = 60 \times ? \end{array}$$

$$\begin{array}{l} 14. \quad 8 \times ? = 64 \\ 8 \times ? = 640 \\ \underline{?} \times 80 = 640 \\ 80 \times ? = 6400 \end{array}$$

Compute.

$$15. \quad 70 \overline{)560}$$

$$16. \quad 8 \overline{)720}$$

$$17. \quad 80 \overline{)4800}$$

$$18. \quad 90 \overline{)630}$$

$$19. \quad 700 \overline{)3500}$$

$$20. \quad 60 \overline{)4200}$$

$$21. \quad 70 \overline{)5600}$$

$$22. \quad 80 \overline{)5600}$$

$$23. \quad 80 \overline{)720}$$

$$24. \quad 30 \overline{)480}$$

$$25. \quad 200 \overline{)1800}$$

$$26. \quad 30 \overline{)2700}$$

Compute.

$$1. \quad 3 \overline{)108}$$

$$2. \quad 7 \overline{)229}$$

$$3. \quad 4 \overline{)376}$$

$$4. \quad 6 \overline{)424}$$

$$5. \quad 9 \overline{)558}$$

$$6. \quad 6 \overline{)443}$$

$$7. \quad 8 \overline{)764}$$

$$8. \quad 5 \overline{)350}$$

$$9. \quad 7 \overline{)502}$$

$$10. \quad 15 \overline{)395}$$

$$11. \quad 17 \overline{)362}$$

$$12. \quad 12 \overline{)502}$$

$$13. \quad 11 \overline{)572}$$

$$14. \quad 18 \overline{)562}$$

$$15. \quad 13 \overline{)286}$$

$$16. \quad 16 \overline{)338}$$

$$17. \quad 14 \overline{)462}$$

$$18. \quad 24 \overline{)731}$$

Solve each equation, then answer the question.

19. Nora is packing cookies in boxes. Each box will hold 9 cookies.
How many boxes can she fill with 105 cookies?

Equation: $105 = 9q + r$

20. Fred is pasting baseball cards into his scrapbook. If each page holds 16 cards, how many pages can he fill with 351 cards?

Equation: $351 = 16q + r$

We have learned that in the set of whole numbers, the equation for remainder division is

$$b = aq + r,$$

where b is the dividend, a is the divisor, q is a partial quotient, and r is a remainder.

$$\begin{array}{r} a \longrightarrow 17 \\ \overline{)359} \\ 340 \\ \hline 19 \\ 17 \\ \hline r \longrightarrow 2 \end{array} \quad \begin{array}{r} b \\ | \\ 20 \\ | \\ 1 \\ \hline q \end{array}$$

Check: Does $21 \times 17 + 2 = 359$?

$$\begin{array}{r} 21 \\ \times 17 \\ \hline 357 \\ + 2 \\ \hline 359 \end{array}$$

Compute each exercise, then check your results.

1. $21 \overline{)826}$

2. $19 \overline{)395}$

3. $15 \overline{)357}$

4. $13 \overline{)223}$

5. $16 \overline{)547}$

6. $18 \overline{)957}$

7. $23 \overline{)1039}$

8. $17 \overline{)801}$

Solve each equation, then check your results.

9. $496 = 12q + r$

10. $448 = 31q + r$

11. $517 = 15q + r$

12. $826 = 41q + r$

13. $3192 = 50q + r$

14. $1293 = 62q + r$

15. $2708 = 43q + r$

16. $5102 = 67q + r$

17. $2136 = 27q + r$

18. $1794 = 31q + r$

19. $1307 = 31q + r$

20. $2704 = 53q + r$

Copy and complete each exercise.

1. 47

$$\begin{array}{r} 9550 \\ \times 47 \\ \hline 141 \\ 380 \\ \hline 9550 \end{array}$$

200

Check:

$$\begin{array}{r} \square\square\square \\ \times 47 \\ \hline 9541 \\ + 9 \\ \hline 9550 \end{array}$$

2. 34

$$\begin{array}{r} 2010 \\ \times 47 \\ \hline 14 \\ 80 \\ \hline 90 \end{array}$$

50

3. 15

$$\begin{array}{r} 1576 \\ \times 47 \\ \hline 100 \\ 75 \\ \hline 90 \end{array}$$

100

4. 8

$$\begin{array}{r} 936 \\ \times 8 \\ \hline 72 \\ 10 \\ \hline 56 \end{array}$$

100

10

5. 22

$$\begin{array}{r} 5124 \\ \times 44 \\ \hline 176 \\ 204 \\ \hline 22 \end{array}$$

□□□

30

2

6. 14

$$\begin{array}{r} 3320 \\ \times 14 \\ \hline 132 \\ 30 \\ \hline 200 \end{array}$$

200

□□

7

2□□

7. 37

$$\begin{array}{r} 4586 \\ \times 37 \\ \hline 316 \\ 154 \\ \hline 100 \end{array}$$

□□

3

Compute.

$$1. \ 27 \overline{)213}$$

$$2. \ 28 \overline{)971}$$

$$3. \ 54 \overline{)3921}$$

$$4. \ 67 \overline{)3912}$$

$$5. \ 29 \overline{)2761}$$

$$6. \ 11 \overline{)659}$$

$$7. \ 43 \overline{)3972}$$

$$8. \ 76 \overline{)3957}$$

$$9. \ 63 \overline{)7929}$$

$$10. \ 49 \overline{)7051}$$

$$11. \ 17 \overline{)8059}$$

$$12. \ 37 \overline{)752}$$

$$13. \ 6 \overline{)361}$$

$$14. \ 7 \overline{)492}$$

$$15. \ 13 \overline{)1690}$$

$$16. \ 15 \overline{)2251}$$

$$17. \ 16 \overline{)2560}$$

$$18. \ 25 \overline{)6251}$$

$$19. \ 30 \overline{)9002}$$

$$20. \ 11 \overline{)1216}$$

$$21. \ 59 \overline{)6030}$$

$$22. \ 18 \overline{)1965}$$

$$23. \ 9 \overline{)8128}$$

$$24. \ 13 \overline{)2695}$$

$$25. \ 33 \overline{)1111}$$

$$26. \ 14 \overline{)4321}$$

$$27. \ 36 \overline{)21601}$$

Compute the greatest quotient for each equation. Check your results.

1. $354 = 17q + r$

2. $1496 = 25q + r$

3. $5997 = 37q + r$

4. $3891 = 32q + r$

5. $7926 = 80q + r$

6. $4993 = 72q + r$

Write an equation for each story. Answer each question.

7. Mary reads 18 pages of a 732-page library book each time she has a free reading period. How many free reading periods will Mary need to read the book?
8. An airline company operates 7532 flights every 2 weeks. If the same number of flights are scheduled each day, how many flights are scheduled for 1 day?
9. Each year Mrs. Warner bakes cookies to give as Christmas gifts. This year she baked 2544 cookies. If each of her 35 friends received the same number of cookies, how many cookies did each friend receive?
10. Bay City expects to register 12,400 pupils for school in September. If the school principals want 28 pupils in each classroom, how many classrooms will they need?

Compute the greatest quotient for each equation. Check your results.

1. $396 = 15q + r$

2. $991 = 81q + r$

3. $4962 = 70q + r$

4. $3912 = 18q + r$

5. $857 = 25q + r$

6. $3291 = 16q + r$

Write an equation to show the number structure of each story. Write a sentence to answer each story question.

7. Mr. Brown ordered 432 bottles of soda pop for his grocery store. If each case holds 12 bottles, how many cases did he order?

8. Nancy can drive 239 miles in her car on 11 gallons of gasoline. How many miles can she drive on 1 gallon of gas?

9. On their vacation, the Holmes family traveled 739 miles at an average rate of 35 miles an hour. How many hours did it take them to drive the 739 miles?

10. A printing press produced 6072 books during one 12-hour period. How many books did the press print in one hour?

12 GEOMETRY: LINES

On your paper, mark a point A. Use your ruler to draw several lines through point A.

A •

1. Can you draw another line through point A?
2. Mark point B and point C on your paper. Draw as many lines as you can that pass through both point B and point C. (The point is the exact center of the dot.)
3. How many lines did you draw?
4. Can you draw another line through both point B and point C?
5. Name 3 lines in your classroom by giving the location of 2 points on each line.
6. Copy this diagram. Draw a line through F that *intersects* line DE. Can you find a pair of intersecting lines in your classroom?

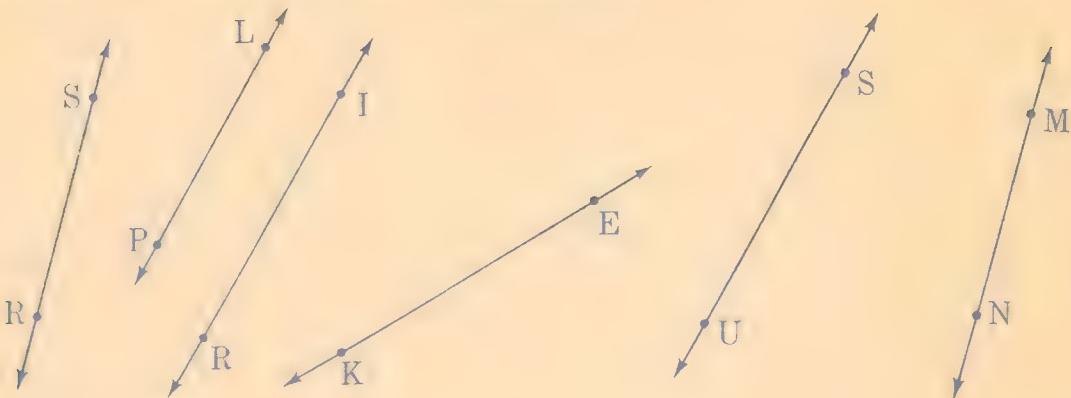
F •



7. Now draw a line through F that is *parallel* to line DE. Can you find a pair of parallel lines in your classroom?
8. Can you draw a line through F that is *skew* to line DE? Why? Can you describe a pair of skew lines in your classroom?
9. Line HI and line JK are parallel. Why? Name all the pairs of parallel lines you can.



Use your compass or ruler to determine which lines in this picture are parallel. Then list each pair of parallel lines.



On your paper, draw line BC.



Could you draw a different line that contains both point B and point C?

Draw a line that intersects line BC. How many points do the two lines have in common?

Could the 2 lines intersect in more than 1 point?

Could you draw a line so that your answer would be different? Why not?

Two distinct lines have either points or points in common.

Many interesting designs are formed by intersecting lines. For example, highways intersect where 2 roads cross. Can you name 4 other examples of intersecting lines?



Two lines are skew or parallel, or they intersect.

Skew lines are lines that are not parallel, and do not intersect.
They do not lie in the same plane.

Parallel lines are lines that are everywhere the same distance apart.
They lie in the same plane.

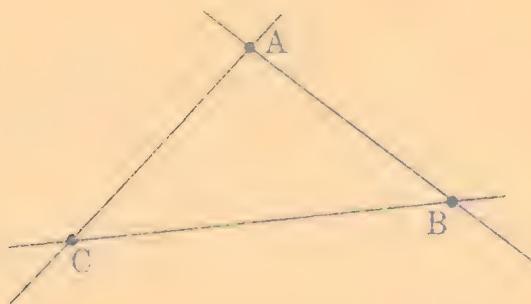
Intersecting lines are lines that intersect in a point.
They lie in the same plane.

1. On your paper, draw 3 lines that intersect in 0 points.
(Hint: They will be parallel.)
2. Draw 3 lines that intersect in 1 point.
3. Draw 3 lines that intersect in 2 points.
(Hint: Are 2 of the lines parallel?)
4. Draw 3 lines that intersect in 3 points.
5. If you can, draw 3 lines that intersect in 4 points.
6. What is the greatest number of points of intersection of 3 lines?

Let us try to find the greatest number of points of intersection that can be determined by a given number of lines. Remember, the greatest number of points of intersection occurs when every pair of lines has its own point of intersection.

7. Draw 2 lines that intersect each other.
The greatest number of points determined by 2 lines is .
8. Draw a line that intersects the two lines in different points.
The greatest number of points determined by 3 lines is $1 + \underline{\quad} = \underline{\quad}$.
9. Draw a line that intersects the three lines in different points.
The greatest number of points determined by lines is $1 + 2 + \underline{\quad} = \underline{\quad}$.
10. Draw a line that intersects all four lines in different points.
The greatest number of points determined by lines is $1 + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$.
11. The greatest number of points determined by 6 lines is .
12. The greatest number of points determined by 7 lines is .
13. The greatest number of points determined by 11 lines is .
14. The greatest number of points determined by 15 lines is .

Copy the figure below. Then complete each exercise.



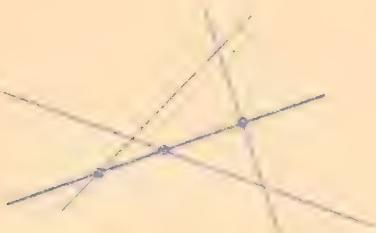
1. With your pencil, trace along line CB. How many points of intersection did you cross?
2. With your pencil, trace along line CA. How many points of intersection did you cross?
3. With your pencil, trace along line AB. How many points of intersection did you cross?
4. Each pair of lines intersects in point.
5. There are lines in the figure, and each line intersects the other 2 lines in points. We have counted $3 \times \underline{\quad}$ points.
6. In the figure, there are points of intersection.
7. When you computed, you counted points of intersection.
8. Why did your computation give 6 points of intersection instead of 3?

Can you use this information to arrive at a method for finding the greatest number of points of intersection determined by 3 lines?

9. To find the greatest number of points of intersection determined by 3 lines, we can multiply 3 by , then divide by .

Do you think you could use this method to find out how many points of intersection 4 lines determine? 5 lines? n lines?

How can we find the greatest number of points of intersection determined by 4 lines?



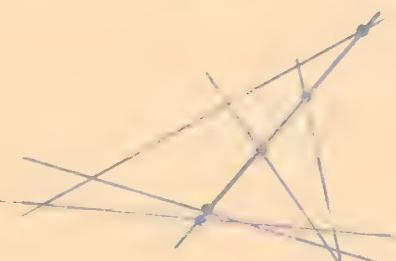
1. In this figure, there are lines.
2. The heavy line intersects each of the other lines in point(s).
3. Does each line intersect every other line in the same number of points?
4. To find the greatest number of points of intersection determined by 4 lines, we multiply the number of lines, , by the number of points of intersection on each line, , and then divide by . We find that there are $\frac{? \times ?}{2}$, or points. Count the points to see if this is true.

Try this with 5 lines.



5. In this figure, there are lines.
6. Each line intersects every other line in point(s).
7. The greatest number of points of intersection determined by 5 lines is $\frac{? \times ?}{2}$, or points.

Now try this with 6 lines.



8. The greatest number of points of intersection determined by 6 lines is $\frac{? \times ?}{2}$, or points.

The greatest number of points of intersection that n lines can determine is $1 + 2 + 3 + \dots + (n - 1)$, or $\frac{n(n - 1)}{2}$. Thus 13 lines can determine a maximum of $1 + 2 + 3 + \dots + 12$, or $\frac{13 \times 12}{2}$, or 78 points of intersection.

Copy and complete the table. Use the addition formula and the multiplication formula to determine the greatest number of points of intersection for each set of lines.

Number of lines	Addition formula	Multiplication formula	Greatest number of points of intersection
2	1	$\frac{2 \times 1}{2}$	1
3	$1 + 2$	$\frac{? \times ?}{2}$	3
4	?	?	?
5	?	?	?
6	?	$\frac{9 \times 8}{2}$?
7	$1 + 2 + 3 + \dots + 10$?	55
8	$1 + 2 + 3 + 4 + 5$?	?
9	?	?	?
10	?	?	?
11	?	?	?



We can use the formula $1 + 2 + 3 + 4 + \dots + (n - 1) = \frac{n(n - 1)}{2}$ to determine the greatest number of points of intersection of a set of n lines.

Complete each sentence.

1. A line is determined by ? points.
2. Two lines are skew or ?, or they ?.
3. Two lines that are each parallel to a third line are ? to each other.
4. Skew lines are lines that do not lie in the same ?. Skew lines are not ? and do not ?.
5. Two lines can intersect in ? or ? point(s).
6. Three lines can intersect in ?, ?, ?, or ? points.
7. The greatest number of points of intersection of a set of 5 lines is $\frac{? \times ?}{?}$ or ?.
8. The greatest number of points of intersection of a set of n lines is $\frac{? \times ?}{2}$.
9. To compute the sum $1 + 2 + 3 + \dots + 7$, we may multiply ? by ? and divide by ?, so $1 + 2 + 3 + \dots + 7 = \underline{?}$.
10. To compute the sum $1 + 2 + 3 + \dots + (n - 1)$, we may multiply ? by ? and divide by ?.

13 MONEY

Write the value of each set of coins. Use the dollar sign and decimal point. If proper, write the value using the cent symbol (¢), too.

1.



2.



3.



4.



The value of money less than 100 cents may be written in dollars or in cents (\$.36 or 36¢). The dollar sign (\$) is always used for values greater than 99¢.

Record the value of each set of bills and coins named in the table.

	Dollars	Half-dollars	Quarters	Dimes	Nickels	Cents	Total Value
1.	4	1	1	2	1	2	?
2.	24	2	3	3	0	2	?
3.	2	4	8	20	40	10	?
4.	100	1	2	0	0	1	?
5.	0	5	2	10	20	0	?
6.	0	0	0	0	0	1713	?
7.	99	1	1	1	2	4	?
8.	69	20	40	60	10	50	?
9.	8	8	8	8	8	8	?
10.	1	2	3	4	5	6	?

Compute each sum.

11. \$4.00

.50

.25

.20

.05

+ .02

12. \$24.00

1.00

.75

.30

+ .02

13. \$2.00

2.00

2.00

2.00

+ .10

14. \$100.00

.50

.50

.01

+ .01

15. \$2.50

.50

1.00

+ 1.00

16. \$10.00

7.00

.10

+ .03

17. \$99.00

.50

.25

.10

+ .04

18. \$69.00

10.00

10.00

6.00

+ .50

19. \$8.00

4.00

2.00

.80

+ .80

20. \$1.00

1.00

.75

.40

+ .25

Make a chart and list all of the United States coins and bills in current use. Then write the value of each coin and bill.

Coin	Value	Bill	Value
cent	1¢	?	\$1
nickel	?	2-dollar	?
dime	?	?	?
quarter	?	10-dollar	?
half-dollar	?	?	?
silver dollar	?	?	\$50
		?	\$100
		?	\$500
		?	\$1,000
		?	\$5,000
		?	\$10,000

Name bills and coins that could be used for each amount of money named. Try to name the least possible number of bills and coins.

\$377.04 7 bills and 4 coins

1. \$11,111.11 ? bills and ? coins
2. \$24,392.15 ? bills and ? coins
3. \$100,000.53 ? bills and ? coins
4. \$9,999.99 ? bills and ? coins
5. \$16,688.91 ? bills and ? coins

Use the symbol \$ or ¢ to express each amount named.

- | | | | |
|----------------|---------------|---------------|----------------|
| 6. 17 cents | 7. 8 cents | 8. 298 cents | 9. 59 cents |
| 10. 10 cents | 11. 402 cents | 12. 6 cents | 13. 21 cents |
| 14. 1567 cents | 15. 103 cents | 16. 322 cents | 17. 1906 cents |

Rewrite each exercise so that the amounts are named in order from greatest to least.

1. \$11.04, 11 dollars and 40 cents, 14 cents, 104 cents
2. \$6.91, 60 dollars and 19 cents, 69 cents, 619 cents, \$6.09
3. \$.80, 180 cents, 18 cents, \$18.00, eighty dollars

Write and solve an addition equation to compute the sum in each exercise. Use dollar signs and decimal points for each amount of money named.

4. 8 dimes, 5 half-dollars, 3 nickels, 3 quarters, 9 pennies
5. 7 dimes, 3 half-dollars, 4 pennies, 2 nickels, 9 quarters
6. 5 dollars, 1 nickel, 6 dimes, 5 quarters
7. 2 cents, 9 half-dollars, 3 silver dollars, 2 two-dollar bills
8. 19 cents, 4 half-dollars, 3 quarters, 1 nickel, 1 penny
9. 3 half-dollars, 7 pennies, 2 nickels, 5 dimes
10. 1 dollar, 1 quarter, 1 nickel, 11 pennies, 2 dimes
11. 3 two-dollar bills, 1 quarter, 7 dimes, 3 nickels
12. 2 one-dollar bills, 3 quarters, 4 dimes, 5 nickels

Find and write the answer to each story question.

1. Ellen went shopping for her mother. She bought a loaf of bread for 21¢, a quart of milk for 24¢, a dozen eggs for 56¢, and a steak for \$1.14. How much did Ellen spend?
2. Kirk's father bought him a baseball bat for \$2.75 and a baseball for \$3.10. How much did he spend?
3. Alice earned \$12.15 during the summer. She bought a fall skirt for \$8.47. How much money did she have left?
4. The fourth grade had a bake sale to raise money. They wanted to buy a record that cost \$4.75 and a film strip that cost \$6.50. After the sale, the class counted 2 one-dollar bills, 4 half-dollars, 7 quarters, 26 dimes, 43 nickels, and 9 pennies. Did they make enough to buy the record and the film strip?

Compute. Be sure to show each sum or difference as an amount of money.

$$\begin{array}{r} \text{5. } \$5.38 \\ + 6.54 \\ \hline \end{array}$$

$$\begin{array}{r} \text{6. } \$7.24 \\ + 2.89 \\ \hline \end{array}$$

$$\begin{array}{r} \text{7. } \$5.00 \\ + 2.89 \\ \hline \end{array}$$

$$\begin{array}{r} \text{8. } \$10.00 \\ - 5.66 \\ \hline \end{array}$$

$$\begin{array}{r} \text{9. } \$1.69 \\ + 4.75 \\ \hline \end{array}$$

$$\begin{array}{r} \text{10. } \$14.00 \\ - 2.08 \\ \hline \end{array}$$

$$\begin{array}{r} \text{11. } \$18.00 \\ - .01 \\ \hline \end{array}$$

$$\begin{array}{r} \text{12. } \$97.60 \\ - 8.38 \\ \hline \end{array}$$

$$\begin{array}{r} \text{13. } \$27.00 \\ - 26.95 \\ \hline \end{array}$$

$$\begin{array}{r} \text{14. } \$30.40 \\ - .41 \\ \hline \end{array}$$

$$\begin{array}{r} \text{15. } \$42.90 \\ - 17.63 \\ \hline \end{array}$$

$$\begin{array}{r} \text{16. } \$83.70 \\ + 26.48 \\ \hline \end{array}$$

Complete this newspaper advertisement.

CLEARANCE SALE		CLEARANCE SALE	
Electric Train Set	Ping-Pong Set	Model Airplane	Camera
Was \$7.98	Was \$4.75	Was \$2.98	Was \$13.65
Now \$6.99	Now \$3.98	Now \$1.79	Now <u>D</u>
Save <u>A</u>	Save <u>B</u>	Save <u>C</u>	Save \$2.77
Tool Chest		Sailboat	Doll
Was \$3.85	Was <u>F</u>	Was \$6.50	Was <u>H</u>
Now <u>E</u>	Now \$2.47	Now \$5.99	Now \$19.98
Save \$.27	Save \$.38	Save <u>G</u>	Save \$1.52

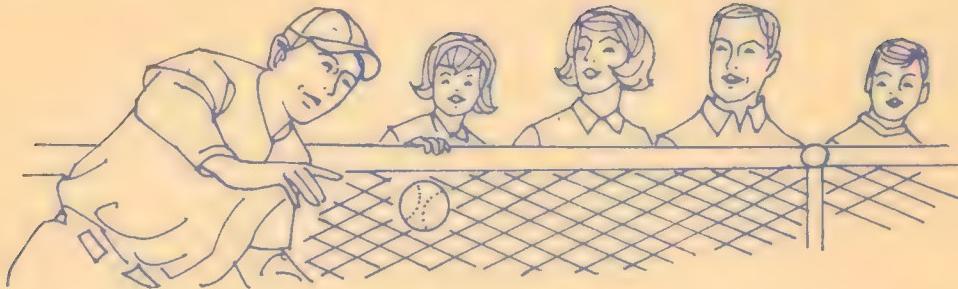
Compute each amount.

$$\begin{array}{r} 1. \quad \$.68 \\ + .25 \\ \hline \end{array} \quad \begin{array}{r} 2. \quad \$.47 \\ + .89 \\ \hline \end{array} \quad \begin{array}{r} 3. \quad \$1.09 \\ + .45 \\ \hline \end{array} \quad \begin{array}{r} 4. \quad \$4.35 \\ + 1.66 \\ \hline \end{array} \quad \begin{array}{r} 5. \quad \$8.14 \\ + 6.98 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad \$.43 \\ - .27 \\ \hline \end{array} \quad \begin{array}{r} 7. \quad \$1.05 \\ - .68 \\ \hline \end{array} \quad \begin{array}{r} 8. \quad \$1.29 \\ - .84 \\ \hline \end{array} \quad \begin{array}{r} 9. \quad \$7.15 \\ - 2.47 \\ \hline \end{array} \quad \begin{array}{r} 10. \quad \$12.48 \\ - 9.53 \\ \hline \end{array}$$

$$\begin{array}{r} 11. \quad \$15.75 \\ + 4.95 \\ \hline \end{array} \quad \begin{array}{r} 12. \quad \$20.00 \\ - 4.17 \\ \hline \end{array} \quad \begin{array}{r} 13. \quad \$15.05 \\ - 4.99 \\ \hline \end{array} \quad \begin{array}{r} 14. \quad \$ 3.75 \\ + 14.88 \\ \hline \end{array} \quad \begin{array}{r} 15. \quad \$10.00 \\ - 9.33 \\ \hline \end{array}$$

Complete each story exercise.



1. Last Sunday the Carson family went to a baseball game. Mr. and Mrs. Carson's tickets cost \$2.85 each, and Marla's and Jeff's tickets cost \$1.70 each. The four tickets cost .
2. Mr. Carson gave the ticket seller a \$10 bill. He received change.
3. The ticket seller counted out the change one coin at a time. First he repeated the cost of the four tickets; then, as he counted each coin, he said the new total. The ticket seller said, " , \$9.20, , , \$10."
4. Suppose the ticket seller had only nickels and quarters in his cash register. Then he would have counted, " , \$9.15, , , \$9.50, , \$10."
5. During the game, Mr. Carson bought 6 hot dogs at \$.35 each, 5 cups of root beer for a total of \$.60, 2 ice cream bars at \$.10 each, a bag of popcorn for \$.15, and a bag of potato chips for \$.10. The refreshments cost .
6. Mr. Carson gave the vendor a \$5 bill. He received change.
7. The vendor counted out the coins one at a time. First, he repeated the cost of the refreshments. " ," he said, "\$3.25, , , and a makes \$5.00."
8. Jeff bought a souvenir baseball for \$2.35. He gave the clerk a \$1 bill and a \$2 bill. The clerk said, "\$2.35 out of . Here's your change: \$2.35, , , ."
9. Jeff checked his change. "\$2.35 from is ," he said, "and plus , plus is , so that's right. Thank you!"

Compute. Answer the questions.

1. Mrs. Wilson planned a party for her fourth grade class. She bought 3 paper tablecloths for \$.79 each. How much did she spend for the tablecloths?
 2. She bought 20 ice cream bars for \$.05 each. How much did the ice cream bars cost?
 3. Mrs. Wilson bought 2 dozen cupcakes at \$.07 each. How much did she spend for the cupcakes?
-

Compute. Be sure to name each product as an amount of money.

$$\begin{array}{r} \$4.15 \\ \times \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} \$2.09 \\ \times \quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} \$8.24 \\ \times \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} \$3.40 \\ \times \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} \$4.09 \\ \times \quad 90 \\ \hline \end{array}$$

$$\begin{array}{r} \$10.03 \\ \times \quad 80 \\ \hline \end{array}$$

$$\begin{array}{r} \$.06 \\ \times \quad 34 \\ \hline \end{array}$$

$$\begin{array}{r} \$1.32 \\ \times \quad 47 \\ \hline \end{array}$$

$$\begin{array}{r} \$9.04 \\ \times \quad 65 \\ \hline \end{array}$$

$$\begin{array}{r} \$12.51 \\ \times \quad 82 \\ \hline \end{array}$$

$$\begin{array}{r} \$16.98 \\ \times \quad 73 \\ \hline \end{array}$$

$$\begin{array}{r} \$29.07 \\ \times \quad 34 \\ \hline \end{array}$$

Compute and check.

1. $5 \overline{) \$7.65}$

2. $2 \overline{) \$8.78}$

3. $7 \overline{) \$6.51}$

4. $5 \overline{) \$37.25}$

5. $6 \overline{) \$9.06}$

6. $3 \overline{) \$45.60}$

7. $6 \overline{) \$67.80}$

8. $9 \overline{) \$74.43}$

9. $12 \overline{) \$88.08}$

10. $7 \overline{) \$64.61}$

11. $8 \overline{) \$89.68}$

12. $5 \overline{) \$71.75}$

13. $7 \overline{) \$126.63}$

14. $9 \overline{) \$139.23}$

15. $11 \overline{) \$178.42}$

Copy and complete the tables.

\times	2	3	4
\$.39	?	?	?
\$.79	?	?	?
\$ 1.45	?	?	?
\$ 3.98	?	?	?
?	\$ 9.00	?	?
\$ 10.00	?	?	?
?	?	?	\$ 62.00
\$ 9.60	?	?	?

\times	5	6	7
\$.51	?	?	?
\$.89	?	?	?
\$ 2.31	?	?	?
\$ 12.71	?	?	?
\$ 3.47	?	?	?
?	?	\$ 25.26	?
?	\$ 46.60	?	?
?	?	?	\$ 54.39

Copy and complete each table.

\times	10	20	30
\$.65	?	?	?
\$.89	?	?	?
?	?	\$33.20	?
\$3.04	?	?	?
?	?	?	\$137.10
\$5.50	?	?	?
\$10.00	?	?	?
\$20.00	?	?	?

\times	5	15	25
\$.95	?	?	?
\$1.40	?	?	?
?	\$9.25	?	?
\$2.75	?	?	?
\$3.40	?	?	?
?	?	\$67.50	?
\$5.25	?	?	?
\$10.00	?	?	?

Answer each story question.

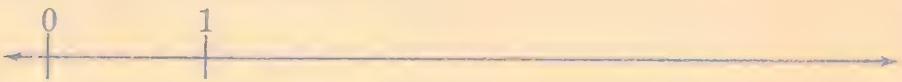
1. Carol asked the postoffice clerk for a dollar's worth of 5¢ stamps. How many stamps did the clerk give Carol?
2. Gayle had \$2.50 to spend on candy for a party. If she bought candy bars that cost \$.10 each, how many candy bars could she buy?
3. Last month Pat made \$3.75 babysitting. If she was paid \$.75 each time, how many times did she babysit last month?
4. Thelma sold some books that she no longer wanted. She sold each book for 25¢, and received a total of \$9.25. How many books did she sell?
5. Bill earned \$35.00 by mowing lawns during the summer. If he mowed 14 lawns and received the same amount for each job, how much did he charge to mow each lawn?

14 INTRODUCTION TO FRACTIONAL NUMBERS

We know that the whole numbers can be mapped on the number line.



The point for 1 is 1 unit-distance from 0.



The point for 2 is ? unit-distances from 0.



The point for 3 is ? unit-distances from 0.



The point for x is half the distance from 0 to 3. It is 3 divided by 2 unit-distances from 0.



There are several ways that we can write 3 divided by 2:

$$\left. \begin{array}{l} 3 \text{ divided by } 2 \\ 3 \div 2 \\ \frac{3}{2} \end{array} \right\} \text{Each of these may be read, "Three divided by two."}$$

On this number line, 2 of these $\frac{3}{2}$ units is $2(\frac{3}{2})$, or ?.



Or, $\frac{3}{2}$ is 1 of the 2 equal parts of 3.

Let's divide 5 by 2.



On this number line, x is 5 divided by ?, or x is $5 \div \underline{\quad}$, or x is $\frac{5}{2}$.

We sometimes read " $\frac{5}{2}$ " as a fraction. We say "five halves," which is the same as saying "five divided by ?."

We can divide a whole number into parts. For example, $\frac{3}{2}$ is part of 3.

It is 3 divided by 2. We can read this as "three divided by two" or as "two halves."

1. $\frac{5}{2}$ is part of 5. It is ? divided by ?. We can read this as "?" or as "?."

Draw this number line and map $\frac{3}{2}$ and $\frac{5}{2}$ onto it.



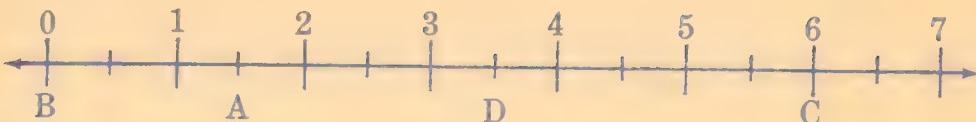
Now complete the number line by mapping more halves.

2. How can you go on beyond $\frac{7}{2}$?
3. List the quotients named on your number line that are whole numbers.
4. List the quotients named on your number line that are not whole numbers.



Any whole number divided by any whole number except 0 is a *quotient*.

Complete each exercise.



1. What number is point A?
2. What number is point B?
3. What number is point C?
4. What number is point D?



5. What number is point P?
6. What number is point Q?
7. What number is point R?
8. What number is point L?
9. Copy the number line below. Then map $\frac{5}{6}$, $2(\frac{5}{6})$, $3(\frac{5}{6})$, $4(\frac{5}{6})$, $5(\frac{5}{6})$, $6(\frac{5}{6})$ onto it.



10. $\frac{5}{6}$ is divided by .

11. $\times (\frac{5}{6}) = 5$

Map 5 onto the number line.

12. The point for 5 is the same as the point for $\times \frac{5}{6}$.



13. Name the points shown on the number line in order.

$$0, \frac{7}{5}, \underline{\quad} \times \frac{7}{5}, \underline{\quad} \times \frac{7}{5}, \underline{\quad} \times \frac{7}{5}, \underline{\quad} \times \frac{7}{5}$$

14. $5 \times \frac{7}{5} = \underline{\quad}$

15. $\frac{7}{5}$ is $7 \div \underline{\quad}$.

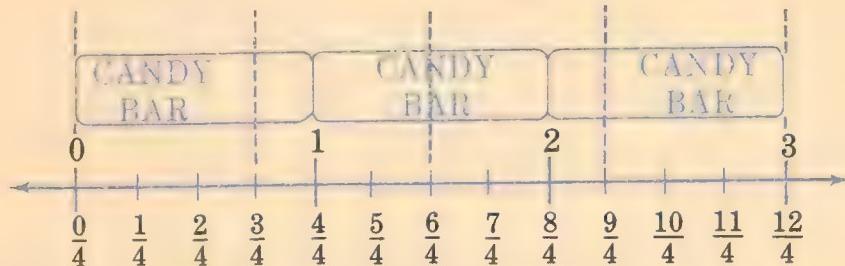
16. $7 \div 5$ is .

17. Five one-fifth parts of seven is .

Judy has 3 candy bars to share with her 3 friends. Each of the 4 girls should have the same amount of candy.

Judy thinks, "There are 4 of us and only 3 candy bars. Each of us will get $\frac{1}{4}$ of the 3 candy bars. How much is that?"

Judy thought a minute. Then she put the candy bars end-to-end and drew a picture and a number line like this:



How many candy bars does each girl get? How do you know?

Judy's friend Alice said, "I would have done it this way." She drew a picture and a number line like Judy's, but she named the points differently.



How many candy bars does each girl get? Is Alice's answer the same as Judy's, or different? Why?

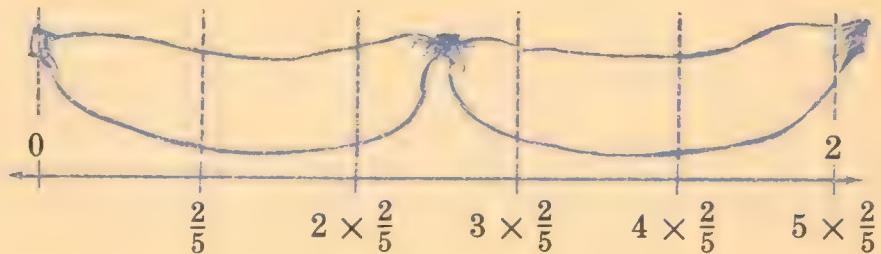
Alice explained that dividing 3 candy bars equally among 4 girls is the same as dividing 3 by 4. "That's what $\frac{3}{4}$ means," she said. "It is the number 3 divided by the number ?."

Each girl received ? candy bars. The four girls had $4 \times \underline{\hspace{1cm}}$, or ?, candy bars.

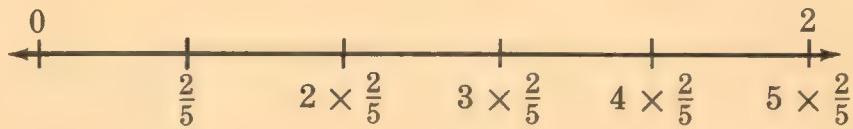
Peter brought 2 bananas to the zoo to feed 5 monkeys. If he divides the bananas equally among the monkeys, how much will he give each monkey?

Peter thought, "I want to divide 2 bananas among 5 monkeys. This means that I want to divide 2 by 5. How many bananas should I give each monkey?"

Peter could draw a picture and a number line like this.



Or he could just draw a number line like this.



Each monkey gets ? bananas. The 5 monkeys have $5 \times \underline{?}$, or ?, bananas.

Would Peter get the same answer if he gave $\frac{1}{5}$ of each banana to each monkey? Why?

If Peter had 3 bananas, how much would each monkey get?

If Peter had 4 bananas, how much would each monkey get?

If Peter had 5 bananas, how much would each monkey get?

To find $\frac{2}{3}$ on this number line, we divide the distance from 0 to 2 into 3 parts. One of these parts is $\frac{2}{3}$. Two of these parts is $2(\frac{2}{3})$.



Complete each exercise.

1. To locate $\frac{7}{4}$, we divide the distance from 0 to 7 into ? parts.

The points of division shown in order are $0, \frac{7}{4}, \underline{?}, \underline{?}, 7$.

2. To locate $\frac{4}{3}$, we divide the distance from 0 to 4 into ? parts.

The points of division shown in order are $0, \frac{4}{3}, \underline{?}, 4$.

3. To locate $\frac{2}{9}$, we divide the distance from 0 to 2 into ? parts.

The points of division shown in order are $0, \frac{2}{9}, \underline{?}, \underline{?}, \underline{?}, \underline{?}, \underline{?}, \underline{?}, \underline{?}, 2$.

4. To locate $\frac{8}{7}$, we divide the distance from 0 to 8 into ? parts.

The points of division shown in order are $0, \frac{8}{7}, \underline{?}, \underline{?}, \underline{?}, \underline{?}, \underline{?}, 8$.

5. To locate $\frac{6}{6}$, we divide the distance from 0 to 6 into ? parts.

The points of division shown in order are $0, \frac{6}{6}, \underline{?}, \underline{?}, \underline{?}, \underline{?}, 6$.

6. To locate $\frac{9}{8}$, we divide the distance from 0 to 9 into ? parts.

The points of division shown in order are $0, \frac{9}{8}, \underline{?}, \underline{?}, \underline{?}, \underline{?}, \underline{?}, \underline{?}, 9$.

7. To locate $\frac{4}{5}$, we divide the distance from 0 to 4 into ? parts.

The points of division shown in order are $0, \frac{4}{5}, \underline{?}, \underline{?}, \underline{?}, 4$.

8. To locate $\frac{5}{9}$, we divide the distance from 0 to 5 into ? parts.

The points of division shown in order are $0, \frac{5}{9}, \underline{?}, \underline{?}, \underline{?}, \underline{?}, \underline{?}, \underline{?}, 5$.

9. To locate $\frac{11}{3}$, we divide the distance from 0 to 11 into ? parts.

The points of division shown in order are $0, \frac{11}{3}, \underline{?}, 11$.

Study the regions. Then complete each exercise.

1. If F is 1, then E is 2,

D is ?, C is ?,

B is ?, A is ?.

2. If F is 2, then E is 4,

D is ?, C is ?,

B is ?, A is ?.

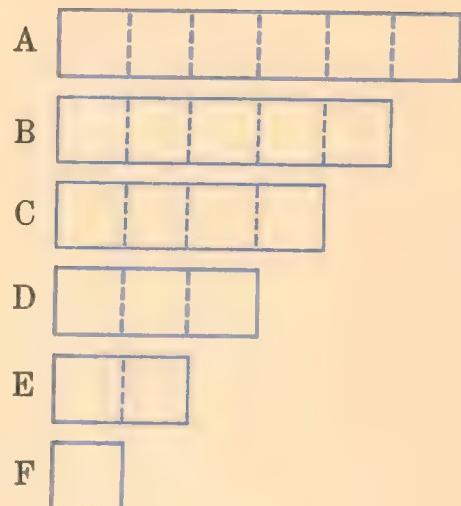
3. If F is $\frac{1}{2}$, then E is $\frac{2}{2}$,

D is ?, C is ?,

B is ?, A is ?.

4. If $\frac{1}{2}$ is F, what is 1?

What is 2? What is 3?



5. If A is 1, then F is $\frac{1}{6}$, E is ?, D is ?, C is ?, B is ?.

6. If A is 2, then F is ?, E is ?, D is ?, C is ?, B is ?.

7. If A is 3, then F is ?, E is ?, D is ?, C is ?, B is ?.

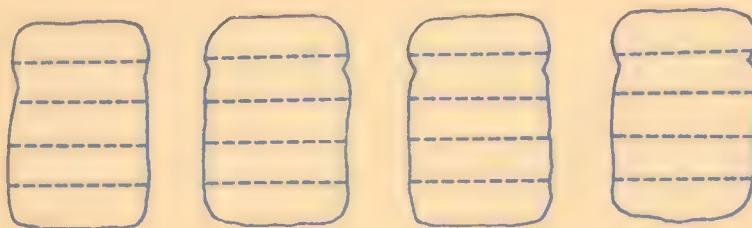
Look at the regions above. In each row of the table, one of the regions has been assigned a value of 1. Complete each row by finding the values of the other regions.

	A	B	C	D	E	F
8.	1	?	?	?	?	?
9.	?	1	?	?	?	?
10.	?	?	1	?	?	?
11.	?	?	?	1	?	?
12.	?	?	?	?	1	?
13.	?	?	?	?	?	1

Tom's club has 5 members. One day, Jim brought 4 sandwiches to the clubhouse. He wanted to share the sandwiches equally with the other members.

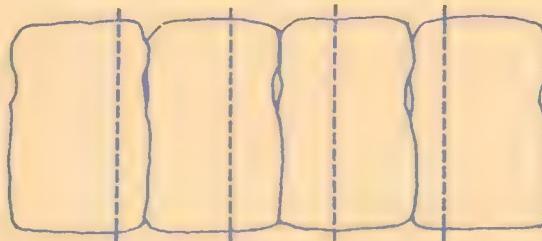
Jim thought, "We have 5 members and only 4 sandwiches. How can I divide them?"

Jim put the sandwiches on a large sheet of paper and drew a diagram like this.



How many sandwiches does each member get? How do you know?

Tom said, "I would have figured it out differently. Look." He put the sandwiches on another sheet of paper and drew this diagram.



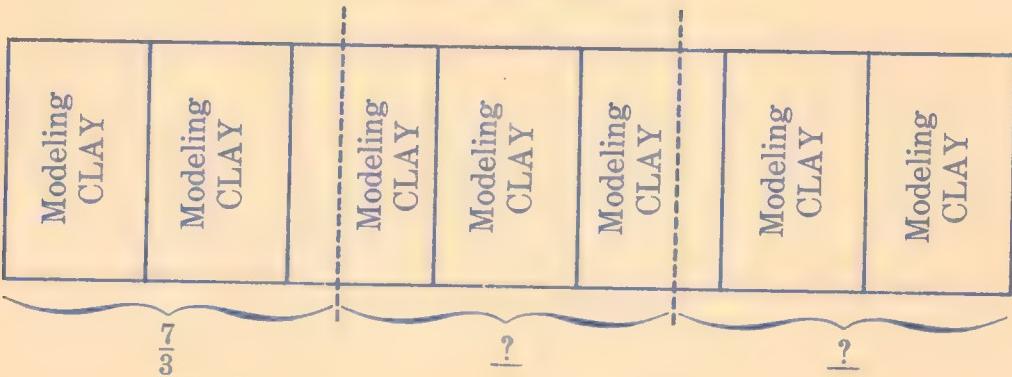
How many sandwiches does each member get? How many sandwiches did each member get last time? Are Jim's and Tom's answers the same or different? Why?

Jim said, "I see, it doesn't make any difference. Dividing 4 sandwiches among 5 club members is dividing 4 by 5. That is what $\frac{4}{5}$ means—the number 4 divided by the number ?."

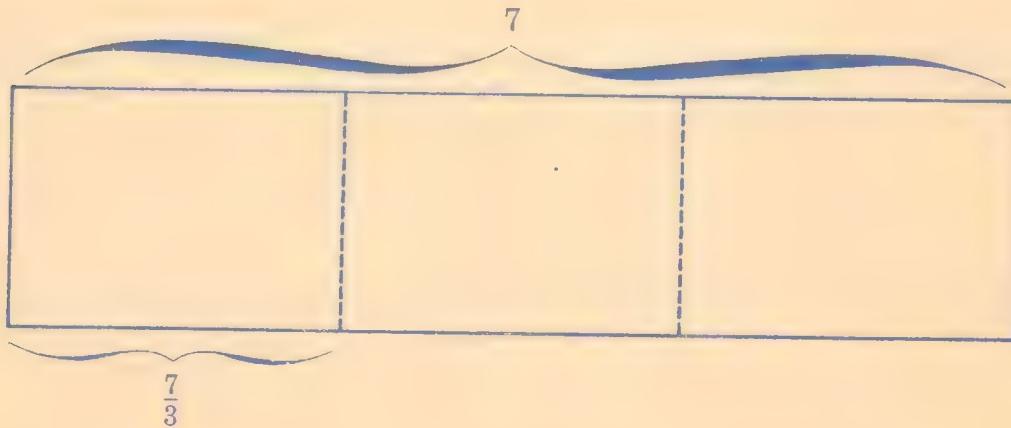
Mrs. Green has 7 boxes of modeling clay. If she divides the clay equally among her 3 children, how many boxes will each child get?

Mrs. Green thinks, "If I divide 7 boxes of clay among 3 children, I am dividing ? by ?. How many boxes of clay should I give to each child?"

Mrs. Green might have divided the clay this way.



Or, she might have drawn this diagram.



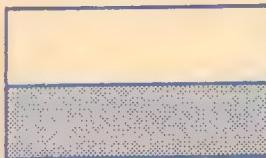
Each child received ? boxes of modeling clay. The 3 children received $3 \times \underline{?}$, or ? boxes of clay.

How would you divide 3 boxes of modeling clay among 7 children?
How much clay does each child get?

The region below is assigned the number 3.

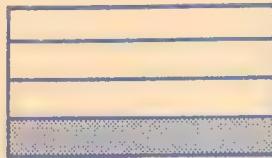


If we partition the region into 2 equal parts, then each part is 3 divided by 2.

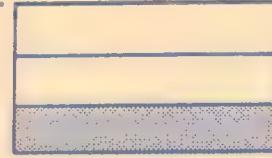


1. The number 3 is assigned to each region below. Write the fractional number for the shaded part of each region.

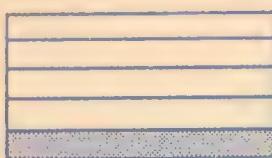
A.



B.



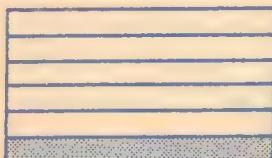
C.



D.



E.



F.



2. Write the fractional number for the unshaded part of each region.

The number 1 is assigned to the region below.



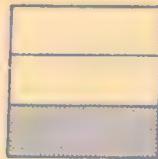
Then we partition the unit-region into 4 equal parts and shade 1 part.



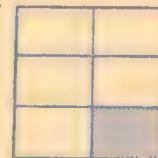
The number for the shaded part is ? divided by ?, or ?.

1. The number 1 is assigned to each region below. Write the fractional number for the shaded part of each region.

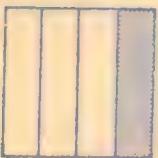
A.



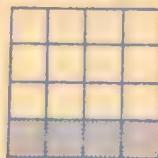
B.



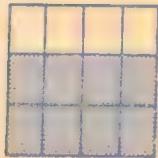
C.



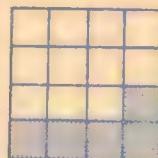
D.



E.

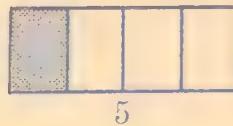


F.



2. Write the fractional number for the unshaded part of each region.

The region below is assigned the number 5.



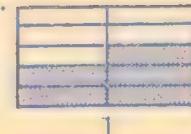
The number for the shaded part is 5 divided by 4, or $\frac{5}{4}$.

1. A number has been assigned to each region below. Write the number for the shaded part of each region.

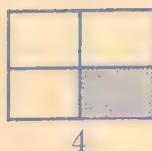
A.



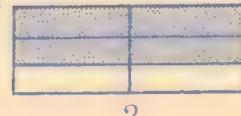
B.



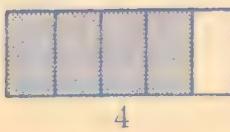
C.



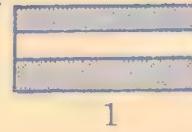
D.



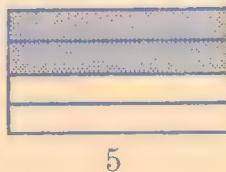
E.



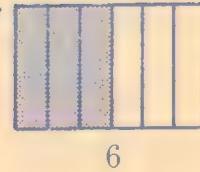
F.



G.



H.



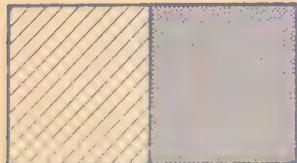
2. Write the fractional number for the unshaded part of each region.

Joan and Betty made a paper kite. Joan colored one half of the kite and Betty colored the other half. Together they colored ? halves of the kite.

$$\frac{1}{2} + \frac{1}{2} = \underline{\underline{2}}$$

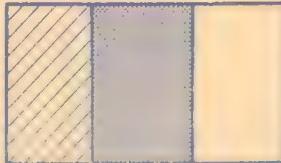
The number 1 is assigned to each region below. Study each region; then copy and complete each equation.

1.



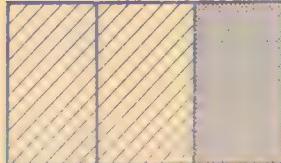
$$\frac{1}{2} + \frac{1}{2} = \underline{\underline{1}}$$

2.



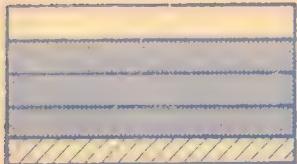
$$\frac{1}{3} + \frac{1}{3} = \underline{\underline{1}}$$

3.



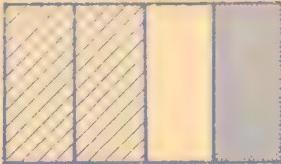
$$\frac{2}{3} + \frac{1}{3} = \underline{\underline{1}}$$

4.



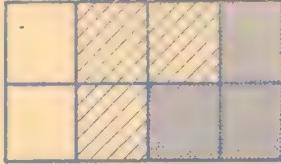
$$\frac{1}{5} + \frac{3}{5} = \underline{\underline{1}}$$

5.



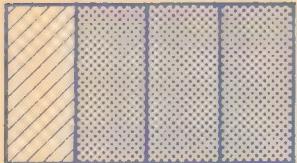
$$\frac{2}{4} + \frac{1}{4} = \underline{\underline{1}}$$

6.



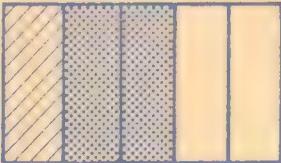
$$\frac{3}{8} + \frac{3}{8} = \underline{\underline{1}}$$

7.



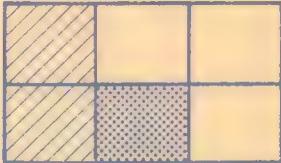
$$\frac{1}{4} + \frac{3}{4} = \underline{\underline{1}}$$

8.



$$\frac{1}{5} + \frac{2}{5} = \underline{\underline{1}}$$

9.



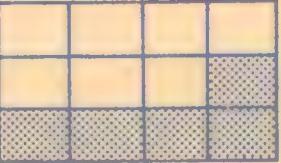
$$\frac{2}{6} + \frac{1}{6} = \underline{\underline{1}}$$

10.



$$\frac{2}{9} + \frac{4}{9} = \underline{\underline{1}}$$

11.



$$\frac{0}{12} + \frac{5}{12} = \underline{\underline{1}}$$

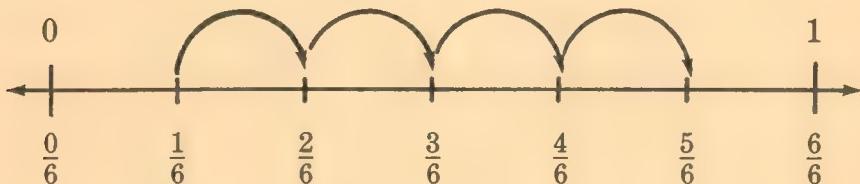
12.



$$\frac{2}{5} + \frac{3}{5} = \underline{\underline{1}}$$

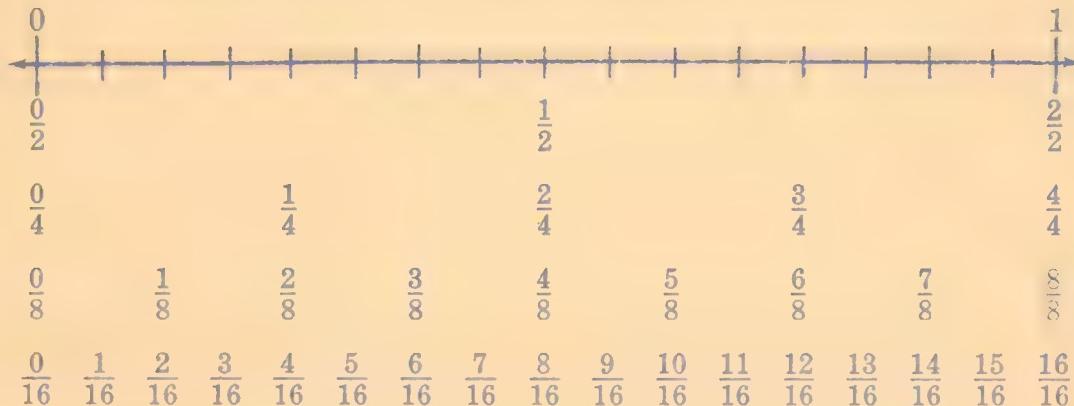
We can use number lines to show addition of fractional numbers.
Study the example.

$$\frac{1}{6} + \frac{4}{6} = ?$$



$$\frac{1}{6} + \frac{4}{6} = \frac{5}{6}$$

Use the number line below to help you compute each sum.



$$1. \frac{2}{8} + \frac{4}{8} = ?$$

$$2. \frac{3}{4} + \frac{2}{4} = ?$$

$$3. \frac{5}{16} + \frac{7}{16} = ?$$

$$4. \frac{6}{8} + \frac{1}{8} = ?$$

$$5. \frac{10}{16} + \frac{5}{16} = ?$$

$$6. \frac{1}{8} + \frac{7}{8} = ?$$

$$7. \frac{2}{4} + \frac{1}{4} = ?$$

$$8. \frac{9}{16} + \frac{2}{16} = ?$$

$$9. \frac{3}{8} + \frac{4}{8} = ?$$

$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

Study the shading in each region. Then copy and complete each pair of equations.



$$\frac{2}{6} + \frac{3}{6} = \underline{\underline{\quad}}$$

$$\frac{5}{6} - \frac{2}{6} = \underline{\underline{\quad}}$$



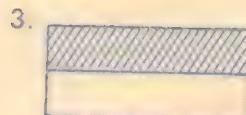
$$\frac{1}{3} + \underline{\underline{\quad}} = \frac{2}{3}$$

$$\frac{2}{3} - \frac{1}{3} = \underline{\underline{\quad}}$$



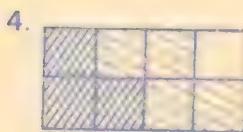
$$\frac{2}{4} + \underline{\underline{\quad}} = \frac{3}{4}$$

$$\frac{3}{4} - \frac{2}{4} = \underline{\underline{\quad}}$$



$$\underline{\underline{\quad}} + \frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{2} - \frac{1}{2} = \underline{\underline{\quad}}$$



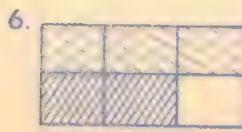
$$\frac{3}{8} + \underline{\underline{\quad}} = \frac{7}{8}$$

$$\frac{7}{8} - \frac{3}{8} = \underline{\underline{\quad}}$$



$$\frac{2}{3} + \underline{\underline{\quad}} = \frac{3}{3}$$

$$\frac{3}{3} - \frac{2}{3} = \underline{\underline{\quad}}$$



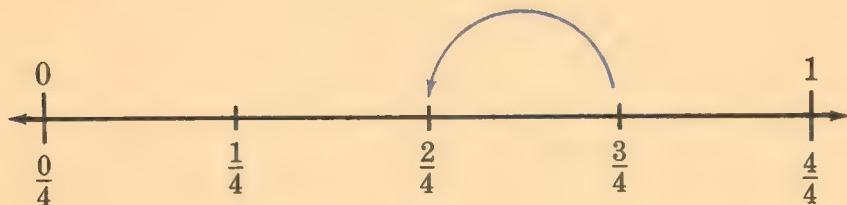
$$\frac{5}{6} - \frac{2}{6} = \underline{\underline{\quad}}$$



$$\frac{6}{6} - \frac{2}{6} = \underline{\underline{\quad}}$$

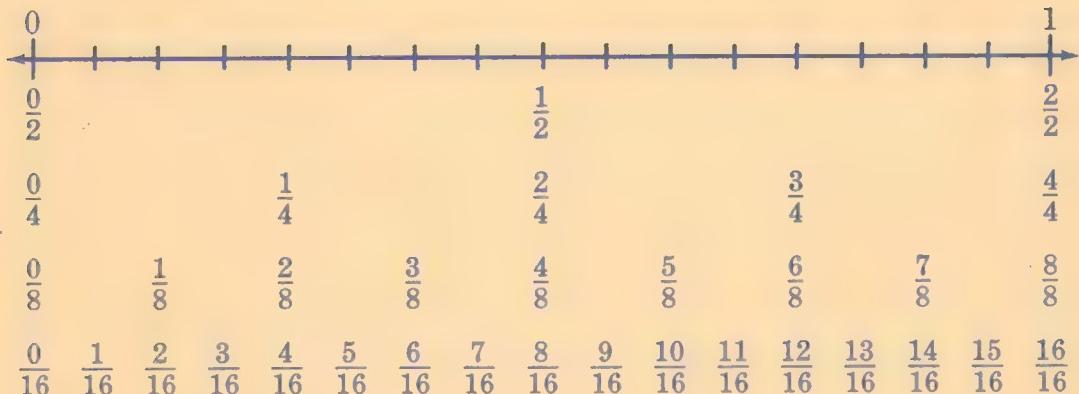
We can show subtraction of fractional numbers on a number line.

$$\frac{3}{4} - \frac{1}{4} = ?$$



$$\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$$

Use the number line below to help you compute each difference.



$$1. \frac{8}{8} - \frac{3}{8} = ?$$

$$2. \frac{2}{4} - \frac{2}{4} = ?$$

$$3. \frac{11}{16} - \frac{5}{16} = ?$$

$$4. \frac{7}{16} - \frac{2}{16} = ?$$

$$5. \frac{3}{8} - \frac{2}{8} = ?$$

$$6. \frac{15}{16} - \frac{11}{16} = ?$$

$$7. \frac{5}{8} - \frac{2}{8} = ?$$

$$8. \frac{4}{4} - \frac{2}{4} = ?$$

$$9. \frac{9}{16} - \frac{7}{16} = ?$$

$$10. \frac{1}{4} - \frac{1}{4} = ?$$

$$11. \frac{7}{8} - \frac{4}{8} = ?$$

$$12. \frac{3}{16} - \frac{2}{16} = ?$$


$$\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

Compute each sum or difference.

1. $\frac{5}{6} + \frac{2}{6}$

2. $\frac{10}{2} + \frac{1}{2}$

3. $\frac{9}{4} - \frac{7}{4}$

4. $\frac{4}{6} + \frac{3}{6}$

5. $\frac{4}{8} - \frac{3}{8}$

6. $\frac{6}{16} - \frac{6}{16}$

7. $\frac{3}{8} + \frac{19}{8}$

8. $\frac{7}{2} + \frac{9}{2}$

9. $\frac{9}{12} + \frac{6}{12}$

10. $\frac{5}{7} - \frac{4}{7}$

11. $\frac{3}{4} + \frac{10}{4}$

12. $\frac{8}{11} + \frac{14}{11}$

Solve each story exercise. Then write a sentence to answer each question.

13. Carol shared 10 candy bars equally among 3 friends and herself.

How many candy bars did each girl get?

14. The girls in art class painted posters. Judy painted $\frac{2}{8}$ of the posters and
_____ painted $\frac{3}{8}$. What part of the posters did the 2 girls paint?

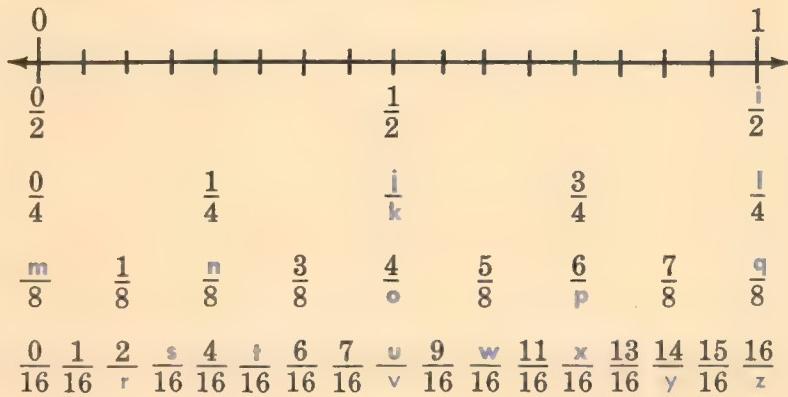
15. George mixed white paint and red paint to make $\frac{3}{4}$ cups of pink
paint. If he used $\frac{1}{4}$ cup of red paint, how much white paint
did he use?

16. Kim used $\frac{1}{3}$ of a pack of pipe cleaners to make a mobile. She used
another $\frac{1}{3}$ of the pack to make a wire design. What part of
the whole pack did Kim use?

17. Ted finished a race in $\frac{7}{12}$ of a minute. John took $\frac{2}{12}$ of a
minute less to finish. How long did John take?

Write the letters from **a** to **z** and after each letter write the number for which it stands.

	1
$\frac{a}{2}$	$\frac{1}{2}$
$\frac{1}{4}$	$\frac{b}{c}$
$\frac{1}{8}$	$\frac{1}{d}$
$\frac{1}{16}$	$\frac{e}{16}$
$\frac{1}{16}$	$\frac{f}{16}$
$\frac{1}{16}$	$\frac{g}{16}$
$\frac{1}{16}$	$\frac{h}{16}$



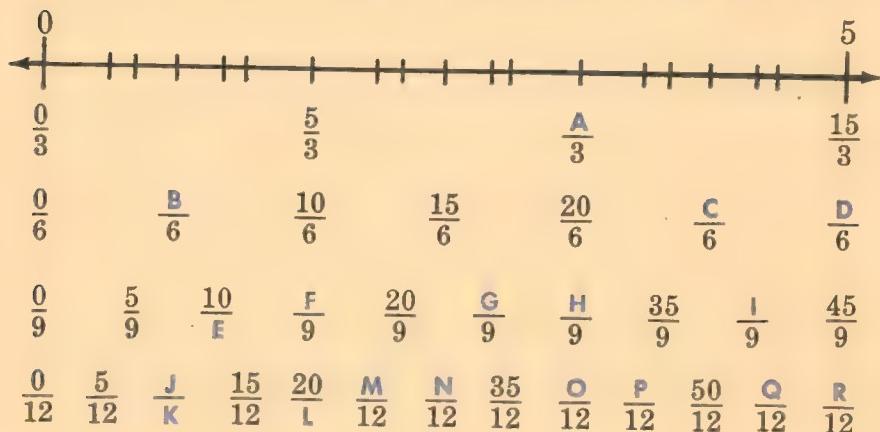
1. Write two fractions for $\frac{1}{4}$.
 2. Write another name for $\frac{2}{16}$.
 3. Write another fraction for $\frac{14}{16}$.
 4. Write three fractions for $\frac{4}{8}$.
 5. Write two other fractions for $\frac{6}{8}$.
 6. Write another fraction for $\frac{6}{16}$.



Fractions that name the same number are *equivalent fractions*.

Write the letters from **a** to **y** and from **A** to **R**, and after each letter write the number for which it stands.

5											
$\frac{5}{3}$		$\frac{5}{a}$		$\frac{5}{b}$							
$\frac{5}{6}$	$\frac{5}{c}$	$\frac{d}{6}$	$\frac{e}{6}$	$\frac{5}{6}$	$\frac{5}{6}$						
$\frac{5}{f}$	$\frac{g}{9}$	$\frac{h}{9}$	$\frac{5}{i}$	$\frac{5}{j}$	$\frac{5}{9}$	$\frac{k}{l}$	$\frac{m}{9}$	$\frac{n}{9}$			
$\frac{5}{12}$	$\frac{5}{o}$	$\frac{p}{12}$	$\frac{q}{r}$	$\frac{5}{12}$	$\frac{s}{t}$	$\frac{u}{12}$	$\frac{5}{12}$	$\frac{5}{v}$	$\frac{5}{w}$	$\frac{x}{y}$	$\frac{5}{12}$

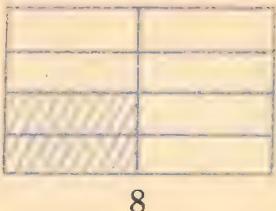


1. Write four fractions for zero.
2. Write four fractions for 5.
3. Write three fractions for $\frac{5}{3}$.
4. Write another name for $\frac{50}{12}$.
5. Write another fraction for $\frac{25}{6}$.
6. Write another fraction for $\frac{5}{6}$.
7. Write three fractions for $\frac{40}{12}$.



Each fractional number can be named by many different equivalent fractions.

The number 8 is assigned to this region.



Write three fractions that name the number for the shaded region.
Different fractions that name the same fractional number are
equivalent fractions.

Complete each row of equivalent fractions.

1. $\frac{1}{4} = \frac{?}{8} = \frac{?}{12}$

2. $\frac{3}{4} = \frac{?}{8} = \frac{12}{?}$

3. $\frac{1}{3} = \frac{?}{6} = \frac{?}{9}$

4. $\frac{4}{3} = \frac{8}{?} = \frac{?}{9}$

5. $\frac{5}{2} = \frac{?}{4} = \frac{?}{8}$

6. $\frac{4}{4} = \frac{?}{8} = \frac{?}{12}$

7. $\frac{7}{4} = \frac{14}{?} = \frac{?}{12}$

8. $\frac{12}{8} = \frac{?}{4} = \frac{?}{2}$

9. $\frac{27}{9} = \frac{?}{3} = \frac{3}{1}$

10. $\frac{6}{12} = \frac{?}{2} = \frac{3}{?}$

11. $\frac{20}{16} = \frac{?}{8} = \frac{?}{4}$

12. $\frac{8}{12} = \frac{16}{?} = \frac{?}{3}$

13. $\frac{3}{3} = \frac{?}{6} = \frac{12}{?}$

14. $\frac{7}{5} = \frac{?}{10} = \frac{70}{?}$

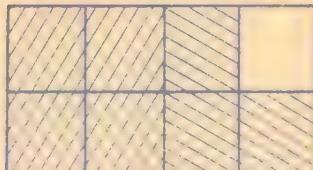
15. Can $\frac{2}{3}$ be expressed as a whole number of sixths? Why?

16. Can $\frac{2}{3}$ be expressed as a whole number of halves? Why?

17. Can $\frac{4}{6}$ be expressed as a whole number of ninths? Why?

How could you compute the sum $\frac{1}{2} + \frac{3}{8}$?

The number 1 is assigned to this region and the region is partitioned into 8 parts. A part of the region is shaded to show $\frac{1}{2}$. Another part is shaded to show $\frac{3}{8}$.



$$\frac{1}{2} + \frac{3}{8} = ?$$

Equivalent fractions make it easy to compute the sum $\frac{1}{2} + \frac{3}{8}$.

The fraction $\frac{1}{2}$ is equivalent to $\frac{?}{8}$, so $\frac{1}{2} + \frac{3}{8} = \frac{?}{8} + \frac{?}{8} = \frac{?}{?}$.

Compute.

1. $\frac{1}{2} + \frac{1}{4} = \frac{?}{4} + \frac{1}{4} = ?$

2. $\frac{2}{4} + \frac{3}{8} = \frac{?}{8} + \frac{3}{8} = ?$

3. $\frac{3}{6} + \frac{1}{3} = \frac{3}{6} + \frac{?}{?} = ?$

4. $\frac{1}{2} + \frac{3}{8} = ?$

5. $\frac{3}{4} + \frac{1}{8} = ?$

6. $\frac{5}{12} + \frac{1}{4} = ?$

7. $\frac{2}{3} + \frac{1}{6} = ?$

8. $\frac{1}{2} + \frac{2}{8} = ?$

9. $\frac{8}{12} + \frac{2}{6} = ?$

10. $\frac{1}{3} + \frac{1}{6} = ?$

11. $\frac{1}{2} + \frac{5}{12} = ?$

12. $\frac{3}{8} + \frac{1}{4} = ?$

13. $\frac{5}{8} + \frac{1}{4} = ?$

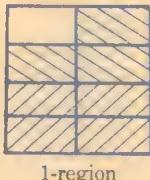
14. $\frac{5}{9} + \frac{1}{3} = ?$

15. $\frac{5}{9} + \frac{2}{3} = ?$

16. $\frac{3}{5} + \frac{9}{10} = ?$

How could you compute the difference $\frac{7}{8} - \frac{1}{2}$?

The number 1 is assigned to this region. Part of the region is shaded to show $\frac{1}{2}$. What part is shaded to show what must be added to $\frac{1}{2}$ to get $\frac{7}{8}$? What number can be assigned to this part?



$$\frac{1}{2} + \underline{\quad} = \frac{?}{8} + \frac{?}{8} = \frac{7}{8}$$

$$\frac{7}{8} - \frac{1}{2} = \frac{7}{8} - \frac{?}{8} = \frac{?}{8}$$

Equivalent fractions make it possible to compute differences of fractional numbers. In subtracting $\frac{2}{3}$ from $\frac{5}{6}$, what fraction for $\frac{2}{3}$ is needed?

$$\frac{5}{6} - \frac{2}{3} = \frac{5}{6} - \frac{?}{6} = \frac{?}{?}$$

Compute.

1. $\frac{1}{3} + \underline{\quad} = \frac{5}{6}$

2. $\frac{1}{2} + \underline{\quad} = \frac{5}{8}$

3. $\frac{3}{5} - \frac{1}{10} = \underline{\quad}$

4. $\frac{1}{2} - \frac{1}{4} = \underline{\quad}$

5. $\frac{3}{4} + \underline{\quad} = \frac{7}{8}$

6. $\frac{3}{5} - \frac{2}{5} = \underline{\quad}$

7. $\frac{6}{6} - \frac{2}{3} = \underline{\quad}$

8. $\frac{2}{3} - \frac{1}{6} = \underline{\quad}$

9. $\frac{3}{4} + \underline{\quad} = \frac{8}{8}$

10. $\frac{3}{8} + \underline{\quad} = \frac{5}{8}$

11. $\frac{5}{6} - \frac{1}{2} = \underline{\quad}$

12. $\frac{1}{2} - \frac{1}{6} = \underline{\quad}$

13. $\frac{3}{4} - \frac{1}{2} = \underline{\quad}$

14. $\frac{5}{8} + \underline{\quad} = \frac{9}{8}$

15. $\frac{7}{4} - \frac{1}{8} = \underline{\quad}$

Compute.

$$1. \frac{4}{3} + \frac{1}{6}$$

$$2. \frac{3}{5} + \frac{4}{5}$$

$$3. \frac{5}{6} - \frac{1}{3}$$

$$4. \frac{5}{4} - \frac{1}{2}$$

$$5. \frac{3}{2} + \frac{7}{2}$$

$$6. \frac{6}{9} - \frac{2}{3}$$

$$7. \frac{7}{8} + \frac{3}{4}$$

$$8. \frac{15}{4} - \frac{3}{2}$$

$$9. \frac{7}{8} - \frac{1}{2}$$

$$10. \frac{5}{8} - \frac{2}{4}$$

$$11. \frac{9}{4} - \frac{3}{2}$$

$$12. \frac{3}{4} + \frac{3}{8}$$

Answer each question.

13. Vickie practiced her trombone $\frac{1}{2}$ hour on Monday and $\frac{3}{4}$ hours on Tuesday. How long did she practice?

14. Paul cut $\frac{1}{3}$ of the lawn before school, and $\frac{5}{12}$ after school. What part of the lawn did Paul cut?

15. In gym class the boys played volleyball first. Then they played dodgeball for $\frac{1}{4}$ of an hour. The games lasted $\frac{1}{2}$ hour. How long did the boys play volleyball?

16. Terry cut a paper strip to make the first letter of her name. She first cut off $\frac{4}{8}$ of the strip, then $\frac{3}{8}$ of the strip. What part of the paper strip did she use?

17. Tom read $\frac{1}{6}$ of his library book on Saturday and $\frac{5}{12}$ of the book on Monday. How much of the book did he read?

18. One week, Ernest spent $\frac{1}{3}$ of his allowance for a birthday gift and $\frac{2}{9}$ for a model airplane. How much of his allowance did he spend?

Compute.

$$1. \frac{35}{2} - \frac{47}{8}$$

$$2. \frac{13}{12} + \frac{5}{4}$$

$$3. \frac{7}{12} - \frac{1}{3}$$

$$4. \frac{9}{6} + \frac{2}{3}$$

$$5. \frac{8}{3} - \frac{4}{9}$$

$$6. \frac{1}{12} + \frac{1}{3}$$

$$7. \frac{2}{3} + \frac{3}{6}$$

$$8. \frac{4}{5} - \frac{1}{10}$$

$$9. \frac{3}{8} + \frac{5}{2}$$

$$10. \frac{28}{3} - \frac{19}{6}$$

$$11. \frac{14}{2} - \frac{5}{4}$$

$$12. \frac{17}{4} + \frac{9}{2}$$

Answer each question.

13. Jane bought $\frac{7}{8}$ yards of material. She used $\frac{3}{4}$ yards for a blouse.
How much material did she have left?

14. Joan's mother bought a ham. She used $\frac{3}{8}$ of the ham to make sandwiches and $\frac{1}{4}$ to make soup. What part of the ham did she use?

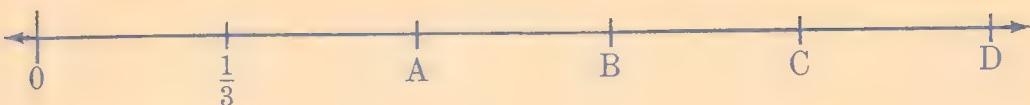
15. Sally made 2 mufflers. She used $\frac{2}{3}$ yards of material for 1 muffler and $\frac{5}{6}$ yards for the other. How many yards of material did she use?

16. Bill walks $\frac{3}{4}$ miles to school every morning, and Francis walks $\frac{5}{8}$ miles. How much farther does Bill walk than Francis?

17. Lou spent $\frac{3}{8}$ hours studying English. Then he spent $\frac{1}{4}$ hour on Math. How much longer did he spend studying English?

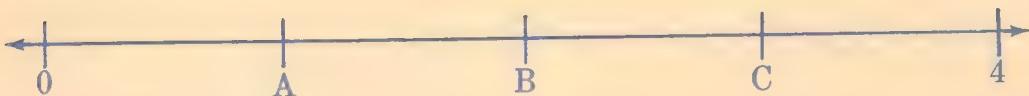
18. Jim helped his father paint their garage. They painted $\frac{1}{3}$ of the garage on Friday and $\frac{5}{9}$ on Saturday. How much of the garage did they paint?

1. What letter on this number line represents 1?



How did you find 1?

2. What letter on this number line represents 1?



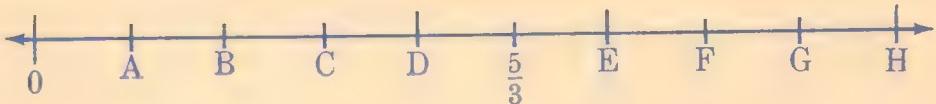
How did you find 1?

3. What letter on this number line represents 1?



How did you find 1?

4. What letter on this number line represents 1?



How did you find 1?

5. What part of 4 is 1?

6. How many $\frac{1}{4}$ parts are in 1?

7. What part of $\frac{5}{2}$ is 1?

8. How many $\frac{2}{5}$ parts are in 1?

9. ? of $\frac{3}{4}$ is 1.

10. ? of $\frac{4}{3}$ is 1.

1. On this number line, what letter represents $\frac{1}{3}$ of 3?



$\frac{1}{3}$ of 3 is ?.

- On this number line, what letter represents $3 \times \frac{1}{3}$?



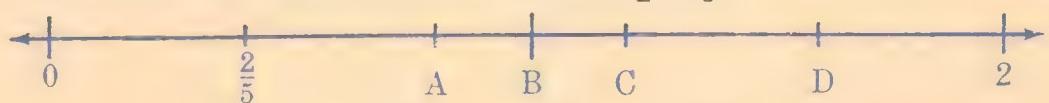
$3 \times \frac{1}{3}$ is ?.

2. On this number line, what letter represents $\frac{2}{5}$ of $\frac{5}{2}$?



$\frac{2}{5}$ of $\frac{5}{2}$ is ?.

- On this number line, what letter represents $\frac{5}{2}$ of $\frac{2}{5}$?



$\frac{5}{2}$ of $\frac{2}{5}$ is ?.

$\frac{5}{2}$ of $\frac{2}{5}$ is $\frac{5}{2} \times \frac{2}{5}$.

$\frac{5}{2} \times \frac{2}{5} = \underline{\hspace{1cm}}$.

3. On this number line, what letter represents $\frac{4}{5} \times \frac{5}{4}$?



$\frac{4}{5} \times \frac{5}{4} = \underline{\hspace{1cm}}$.

4. On this number line, what letter represents $\frac{5}{4} \times \frac{4}{5}$?



$\frac{5}{4} \times \frac{4}{5} = \underline{\hspace{1cm}}$.

1. On this number line, what letter represents $\frac{2}{3} \times \frac{3}{2}$?



$$\frac{2}{3} \times \frac{3}{2} = ?$$

Compute.

2. $\frac{4}{5} \times \frac{5}{4} = ?$

3. $\frac{5}{3} \times \frac{3}{5} = ?$

4. $\frac{3}{7} \times \frac{7}{3} = ?$

5. $\frac{7}{8} \times \frac{8}{7} = ?$

6. $\frac{4}{5} \times ? = 1$

7. $? \times \frac{8}{7} = 1$

8. $\frac{5}{1} \times ? = 1$

9. $4 \times ? = 1$

10. $? \times \frac{16}{21} = 1$

11. $\frac{4}{9} \times ? = 1$

12. $\frac{5}{17} \times ? = 1$

13. $? \times \frac{3}{16} = 1$

If two numbers are multiplied and their product is 1, the numbers are *reciprocals* of each other.

14. Since $\frac{5}{3} \times \frac{3}{5} = ?$; then $\frac{5}{3}$ and $\frac{3}{5}$ are reciprocals. The product $\frac{5}{3} \times \frac{3}{5}$ is 1.

15. The reciprocal of $\frac{3}{8}$ is $?$; of 4 is $?$; of $\frac{17}{3}$ is $?$.

16. What number multiplied by 0 is 1?

17. Does 0 have a reciprocal? Why?

18. In the set of whole numbers, does 5 have a reciprocal? Why?

19. In the set of fractional numbers, does 5 have a reciprocal? Why?



Every fractional number except 0 has a reciprocal.

15 MORE DIVISION OF WHOLE NUMBERS

Study the example below. The equation at the right shows us what happens in the algorithm.

8	937		
	160	20	$8 \times 20 + 777 = 937$
	777		
	640	80	$8 \times 80 + 137 = 777$
	137		
	80	10	$8 \times 10 + 57 = 137$
	57		
	56	7	$8 \times 7 + 1 = 57$
	1	117	

$$8 \times 117 + 1 = 937$$

Copy and complete the equations.

1. 59	683		
	590	10	$59 \times \underline{\quad} + 93 = 683$
	93		
	59	1	$59 \times \underline{\quad} + 34 = 93$
	34	11	

$$59 \times \underline{\quad} + 34 = 683$$

2. 40	998		
	800	20	$40 \times \underline{\quad} + \underline{\quad} = 998$
	198		
	80	2	$40 \times \underline{\quad} + \underline{\quad} = 198$
	118		
	80	2	$40 \times \underline{\quad} + \underline{\quad} = 118$
	38	24	

$$40 \times \underline{\quad} + \underline{\quad} = 998$$

Compute each quotient. Show the equation for each step.

$$3. 9 \overline{)826}$$

$$4. 23 \overline{)8705}$$

$$5. 87 \overline{)4993}$$

Check each result and correct all errors.

$$\begin{array}{r} \text{1. } 70 \longdiv{920} \\ \quad 700 \\ \hline \quad 220 \\ \quad 140 \\ \hline \quad 80 \\ \quad 70 \\ \hline \quad 10 \end{array}$$

10

$$\begin{array}{r} \text{2. } 50 \longdiv{970} \\ \quad 500 \\ \hline \quad 820 \\ \quad 150 \\ \hline \quad 660 \\ \quad 500 \\ \hline \quad 160 \\ \quad 150 \\ \hline \quad 10 \end{array}$$

3

$$\begin{array}{r} \text{3. } 50 \longdiv{658} \\ \quad 500 \\ \hline \quad 358 \\ \quad 300 \\ \hline \quad 58 \\ \quad 50 \\ \hline \quad 8 \end{array}$$

6

3

7

10

1

3

14

$$\begin{array}{r} \text{4. } 30 \longdiv{993} \\ \quad 600 \\ \hline \quad 393 \\ \quad 300 \\ \hline \quad 93 \\ \quad 90 \\ \hline \quad 3 \end{array}$$

20

10

3

$$\begin{array}{r} \text{5. } 20 \longdiv{640} \\ \quad 400 \\ \hline \quad 240 \\ \quad 200 \\ \hline \quad 40 \\ \quad 40 \\ \hline \quad 0 \end{array}$$

200

10

2

$$\begin{array}{r} \text{6. } 40 \longdiv{635} \\ \quad 400 \\ \hline \quad 235 \\ \quad 200 \\ \hline \quad 35 \end{array}$$

10

5

15

Compute and write an equation for your result.

$$\text{7. } 6 \longdiv{85}$$

$$\text{8. } 8 \longdiv{4127}$$

$$\text{9. } 9 \longdiv{8341}$$

$$\text{10. } 30 \longdiv{810}$$

$$\text{11. } 40 \longdiv{329}$$

$$\text{12. } 40 \longdiv{927}$$

$$\text{13. } 60 \longdiv{960}$$

$$\text{14. } 60 \longdiv{438}$$

$$\text{15. } 80 \longdiv{4127}$$

$$\text{16. } 60 \longdiv{4380}$$

$$\text{17. } 12 \longdiv{378}$$

$$\text{18. } 12 \longdiv{3780}$$

Copy and complete each exercise.

16	742	
	160	?
	582	
	160	?
	422	
	160	?
	262	
	160	?
	102	
	32	?
	70	
	32	?
	38	
	32	?
	?	?

16	742	
	320	?
	422	
	320	?
	102	
	80	?
	22	
	16	?
	?	?

16	742	
	640	?
	102	
	96	?
	?	?

Compute the greatest quotient and least remainder. Check your results.

1. $33 \overline{)693}$

2. $29 \overline{)495}$

3. $19 \overline{)756}$

4. $37 \overline{)459}$

Each computation below is a check for a division exercise. Write the division exercise for each check.

5. $\begin{array}{r} 28 \\ \times 32 \\ \hline 16 \\ 40 \\ 240 \\ 600 \\ \hline 896 \\ + 4 \\ \hline 900 \end{array}$

6. $\begin{array}{r} 95 \\ \times 40 \\ \hline 200 \\ 3600 \\ \hline 3861 \end{array}$

7. $\begin{array}{r} 163 \\ \times 7 \\ \hline 21 \\ 420 \\ 700 \\ \hline 1141 \\ + 5 \\ \hline 1146 \end{array}$

8. $\begin{array}{r} 47 \\ \times 81 \\ \hline 47 \\ 560 \\ 3200 \\ 3807 \\ + 15 \\ \hline 3822 \end{array}$



$b = aq + r$ tells us that $b \div a = q$ with a remainder r .

Study and reconstruct each partial check. Complete each division algorithm.

$$\begin{array}{r} 1. \quad 3\ 4 \\ \times \underline{\square\ \square\ \square} \\ 3\ 4\ 6\ 8 \\ + \underline{\quad\ \square} \\ 3\ 4\ 7\ 5 \end{array}$$

$$34 \overline{)3475}$$

$$\begin{array}{r} 2. \quad 7\ 2 \\ \times \underline{\square\ \square} \\ 1\ 0\ 0\ 8 \\ + \underline{\quad\ \square} \\ 1\ 0\ 1\ 0 \end{array}$$

$$72 \overline{)1010}$$

$$\begin{array}{r} 3. \quad 8\ 2 \\ \times \underline{\square\ \square} \\ 2\ 3\ 7\ 8 \\ + \underline{\quad\ \square\ \square} \\ 2\ 3\ 9\ 7 \end{array}$$

$$\begin{array}{r} 4. \quad 1\ 4 \\ \times \underline{\square\ \square} \\ 1\ 2\ 7\ 4 \\ + \underline{\quad\ \square\ \square} \\ 1\ 2\ 8\ 7 \end{array}$$

$$5. \quad 8\ 4$$

$$\begin{array}{r} 6. \quad 5\ 3 \\ \times \underline{\square\ \square} \\ 2\ 0\ 6\ 7 \\ + \underline{\quad\ \square\ \square} \\ 2\ 0\ 9\ 6 \end{array}$$

$$7. \quad 5\ 7$$

$$8. \quad 4\ 9\ 2$$

Answer each question. Show your work.

9. If a bus driver drives 56 miles each hour he is on the road, how many hours on the road will it take him to drive 504 miles?
10. Tom and his family drove 924 miles on their vacation trip. If the car travels 14 miles on each gallon of gasoline, how many gallons did the car use?
11. There are 938 books in the Mead School library. If each of 27 classes wants an equal number of books at the same time, what is the greatest number of books each class can borrow? How many books will remain in the library?

Compute. Study the algorithms to help you answer the questions.

$$1. \ 37 \overline{)926}$$

$$2. \ 37 \overline{)1042}$$

$$3. \ 37 \overline{)1067}$$

$$4. \ 37 \overline{)1383}$$

$$5. \ 37 \overline{)2579}$$

$$6. \ 37 \overline{)25,790}$$

7. What patterns can you find in the divisors?
 8. What patterns can you find in the dividends?
 9. What patterns can you find in the quotients?
 10. What patterns can you find in the remainders?
-

Read each story. Compute to find the solution for each exercise.
Then answer each question.

11. The library at Lakeview School has 738 books. Each shelf holds 23 books. How many shelves are filled? How many books are left over?
12. There are 938 fourth-grade pupils in Central School. How many classes of 29 pupils can be formed? How many pupils will remain for another class?
13. Mary baked 827 cupcakes for a school fair. If she packs 24 cupcakes in a box, how many full boxes will she have? How many cupcakes will remain?
14. Tom has 497 stamps to put in his album. If he puts 18 stamps on each page, how many full pages will he have? How many stamps will remain?

Compute.

$$1. \quad 25 \overline{)9764}$$

$$2. \quad 35 \overline{)9764}$$

$$3. \quad 45 \overline{)9764}$$

$$4. \quad 55 \overline{)9764}$$

$$5. \quad 65 \overline{)9764}$$

$$6. \quad 75 \overline{)9764}$$

$$7. \quad 85 \overline{)9764}$$

$$8. \quad 95 \overline{)9764}$$

$$9. \quad 105 \overline{)9764}$$

$$10. \quad 8 \overline{)488}$$

$$11. \quad 80 \overline{)4880}$$

$$12. \quad 800 \overline{)48800}$$

$$13. \quad 16 \overline{)488}$$

$$14. \quad 160 \overline{)4880}$$

$$15. \quad 1600 \overline{)48800}$$

$$16. \quad 32 \overline{)488}$$

$$17. \quad 320 \overline{)4880}$$

$$18. \quad 3200 \overline{)48800}$$

$$19. \quad 44 \overline{)2401}$$

$$20. \quad 45 \overline{)2401}$$

$$21. \quad 46 \overline{)2401}$$

$$22. \quad 47 \overline{)2401}$$

$$23. \quad 48 \overline{)2401}$$

$$24. \quad 49 \overline{)2401}$$

$$25. \quad 12 \overline{)7548}$$

$$26. \quad 15 \overline{)7548}$$

$$27. \quad 18 \overline{)7548}$$

$$28. \quad 21 \overline{)7548}$$

$$29. \quad 24 \overline{)7548}$$

$$30. \quad 27 \overline{)7548}$$

Compute.

$$1. \ 2 \overline{)2522}$$

$$2. \ 3 \overline{)2523}$$

$$3. \ 4 \overline{)2524}$$

$$4. \ 5 \overline{)2525}$$

$$5. \ 6 \overline{)2526}$$

$$6. \ 7 \overline{)2527}$$

$$7. \ 8 \overline{)2528}$$

$$8. \ 9 \overline{)2529}$$

$$9. \ 10 \overline{)2530}$$

$$10. \ 20 \overline{)120120}$$

$$11. \ 22 \overline{)120120}$$

$$12. \ 24 \overline{)120120}$$

$$13. \ 26 \overline{)120120}$$

$$14. \ 28 \overline{)120120}$$

$$15. \ 30 \overline{)120120}$$

$$16. \ 7 \overline{)27714}$$

$$17. \ 8 \overline{)27715}$$

$$18. \ 9 \overline{)27716}$$

$$19. \ 10 \overline{)27717}$$

$$20. \ 11 \overline{)27718}$$

$$21. \ 12 \overline{)27719}$$

$$22. \ 9 \overline{)7777}$$

$$23. \ 27 \overline{)7777}$$

$$24. \ 81 \overline{)7777}$$

$$25. \ 64 \overline{)6912}$$

$$26. \ 32 \overline{)6912}$$

$$27. \ 16 \overline{)6912}$$

$$28. \ 8 \overline{)6912}$$

$$29. \ 4 \overline{)6912}$$

$$30. \ 2 \overline{)6912}$$

Use what you know about division to help you complete these sentences. Replace the \diamond with $>$, $<$, or $=$ to make each sentence true.

1. $93 \div 2 \diamond 93 \div 3$

2. $176 \div 27 \diamond 203 \div 27$

3. $168 \div 20 \diamond 1680 \div 200$

4. $27 \div 27 \diamond 127 \div 127$

5. $2891 \div 95 \diamond 2892 \div 97$

6. $3210 \div 80 \diamond 321 \div 8$

7. $2891 \div 95 \diamond 2890 \div 93$

8. $267 \div 103 \diamond 267 \div 82$

9. $93 \div 1 \diamond 93 \div 93$

10. $5843 \div 43 \diamond 5843 \div 86$

Answer each question.

11. Dave works 18 hours each week at a supermarket. How many weeks does it take him to work 323 hours?
12. One day he sorted 998 onions into bags of 12 onions. How many bags did he fill?
13. On another day, Dave packed 739 tomatoes into 32 baskets. He put the same number of tomatoes into each basket. How many tomatoes were in each basket?
14. On Friday, Dave put 768 bottles of soft drinks on the shelves. If there were 24 bottles in each case, how many cases did he empty?
15. On Saturday, Dave opened 12 cases of soap. Each case held 72 bars of soap. How many bars of soap were in the 12 cases?

If you know a length in inches, how can you express the same length in feet? (12 inches = 1 foot)

If you know a length in feet, how can you express the same length in yards? (3 feet = 1 yard)

Answer each question.

1. Bob's father built a swimming pool. The length of one side is 288 inches.

What is the length of the side in feet?

What is the length of the side in yards?

2. At the deep end, the pool has a depth of 108 inches.

What is this depth in feet?

What is this depth in yards?

3. Bob painted a red stripe on the four edges of the pool. The stripe has a total length of 864 inches.

What is the length in feet?

What is the length in yards?

4. The diving board has a length of 144 inches.

What is the length in feet?

What is the length in yards?



5. How many feet is 6048 inches? How many yards?

6. How many feet is 62,964 inches? How many yards?

7. How many feet is 34,236 inches? How many yards?

Write an equation to answer each question.

1. The new section of an expressway is 26,400 feet long.
How many yards long is this section of the expressway?
How many miles long is this section of the expressway?
(Hint: 1 mile = 5,280 feet)
2. Jack figured that he walks 36,960 feet each week in going to and from school.
How many miles does he walk each week in going to and from school?
What is this distance in inches?
3. Sue found that the distance around her room is 612 inches.
What is this distance in yards?
What is this distance in feet?
4. Tim said that the distance around his back yard is 240 feet.
What is this distance in yards?
What is this distance in inches?

Compute.

$$5. \quad 221 \overline{)68731}$$

$$6. \quad 226 \overline{)68704}$$

$$7. \quad 231 \overline{)68607}$$

$$8. \quad 5310 \overline{)15930}$$

$$9. \quad 6312 \overline{)37872}$$

$$10. \quad 7324 \overline{)65916}$$

$$11. \quad 8383 \overline{)92213}$$

$$12. \quad 4376 \overline{)96272}$$

$$13. \quad 2898 \overline{)95634}$$

Write a story for each equation.

$$14. \quad 1780 \div 36 = b$$

$$15. \quad 32,765 \div 12 = c$$

$$16. \quad 600,000 \div 5280 = d$$

$$17. \quad 3728 \div 3 = g$$

Check each result to find the dividend. Then reconstruct the whole algorithm.

1.

$$\begin{array}{r} 12 \\ \times 37 \\ \hline 1 \quad 4488 \end{array}$$

2.

$$\begin{array}{r} 321 \\ \times 60 \\ \hline 60 \quad 214 \end{array}$$

3.

$$\begin{array}{r} 4876 \\ \times 19 \\ \hline 1140 \quad 19 \end{array}$$

4.

$$\begin{array}{r} 5035 \\ \times 83 \\ \hline 1737 \quad 83 \end{array}$$

Answer the questions. Show your work.

5. Gherman Titov orbited the earth 17 times in 1961. His ship, *Vostok*, traveled about 4287 hundred miles while in orbit. About how many hundreds of miles did it travel in each orbit?
6. In 1963, L. Gordon Cooper orbited the earth 22 times. His ship, *Faith 7*, traveled about 5564 hundred miles while in orbit. About how many hundreds of miles did it travel in each orbit?
7. It took 91 minutes for *Faith 7* to travel 25,291 miles around the earth. About how many miles did *Faith 7* travel in one minute?
8. An early Mercury capsule reached a height of 55 miles. In March, 1965, Virgil Grissom and John Young orbited the earth 3 times in a Gemini spacecraft. During their first orbit, they reached a height of 140 miles. During their second orbit, they reached a height of 105 miles. How much higher than the Mercury capsule did Grissom and Young go in their first orbit? In their second orbit?

Compute.

$$1. \ 36 \overline{)76}$$

$$2. \ 32 \overline{)96}$$

$$3. \ 24 \overline{)31}$$

$$4. \ 55 \overline{)66}$$

$$5. \ 19 \overline{)95}$$

$$6. \ 19 \overline{)75}$$

$$7. \ 18 \overline{)88}$$

$$8. \ 19 \overline{)76}$$

$$9. \ 66 \overline{)79}$$

$$10. \ 33 \overline{)99}$$

$$11. \ 34 \overline{)90}$$

$$12. \ 14 \overline{)96}$$

$$13. \ 16 \overline{)98}$$

$$14. \ 28 \overline{)94}$$

$$15. \ 21 \overline{)93}$$

$$16. \ 50 \overline{)132}$$

$$17. \ 34 \overline{)259}$$

$$18. \ 78 \overline{)390}$$

$$19. \ 47 \overline{)358}$$

$$20. \ 87 \overline{)270}$$

$$21. \ 93 \overline{)433}$$

$$22. \ 34 \overline{)879}$$

$$23. \ 78 \overline{)468}$$

$$24. \ 24 \overline{)312}$$

Compute.

$$1. \ 60 \overline{)692}$$

$$2. \ 15 \overline{)435}$$

$$3. \ 34 \overline{)633}$$

$$4. \ 64 \overline{)177}$$

$$5. \ 22 \overline{)487}$$

$$6. \ 17 \overline{)544}$$

$$7. \ 12 \overline{)436}$$

$$8. \ 30 \overline{)347}$$

$$9. \ 74 \overline{)373}$$

$$10. \ 37 \overline{)629}$$

$$11. \ 26 \overline{)910}$$

$$12. \ 75 \overline{)964}$$

$$13. \ 75 \overline{)5368}$$

$$14. \ 95 \overline{)9033}$$

$$15. \ 34 \overline{)2210}$$

$$16. \ 42 \overline{)3024}$$

$$17. \ 94 \overline{)9721}$$

$$18. \ 38 \overline{)2204}$$

$$19. \ 67 \overline{)6893}$$

$$20. \ 54 \overline{)7981}$$

$$21. \ 23 \overline{)9900}$$

$$22. \ 16 \overline{)6742}$$

$$23. \ 29 \overline{)9871}$$

$$24. \ 58 \overline{)9791}$$

Compute.

$$1. \quad 98 \overline{)5078}$$

$$2. \quad 73 \overline{)4900}$$

$$3. \quad 88 \overline{)1200}$$

$$4. \quad 213 \overline{)3506}$$

$$5. \quad 472 \overline{)5809}$$

$$6. \quad 116 \overline{)2237}$$

$$7. \quad 880 \overline{)9800}$$

$$8. \quad 220 \overline{)4805}$$

$$9. \quad 110 \overline{)7986}$$

$$10. \quad 725 \overline{)4999}$$

$$11. \quad 832 \overline{)5601}$$

$$12. \quad 458 \overline{)1309}$$

$$13. \quad 700 \overline{)78001}$$

$$14. \quad 640 \overline{)89921}$$

$$15. \quad 254 \overline{)69900}$$

$$16. \quad 852 \overline{)18523}$$

$$17. \quad 611 \overline{)27111}$$

$$18. \quad 213 \overline{)18413}$$

$$19. \quad 547 \overline{)19386}$$

$$20. \quad 365 \overline{)31666}$$

$$21. \quad 298 \overline{)17451}$$

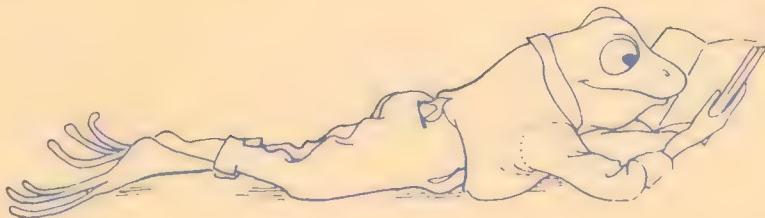
$$22. \quad 999 \overline{)32322}$$

$$23. \quad 888 \overline{)54445}$$

$$24. \quad 808 \overline{)56560}$$

Compute. Then write a sentence to answer each question.

1. An atomic engine runs 14,790 hours on 1 pound of fuel. How many days is this? (One day is 24 hours.)
2. A train carried 241,280 pounds of freight in 13 boxcars. If each boxcar carried an equal weight, how much did each boxcar carry?
3. A machine in the post office sorted the same number of letters each hour for 12 hours. If the machine sorted 65,076 letters, how many did it sort each hour?
4. There are 39,426 books in Midvale School library. If there are 675 students in the school, how many books are there for each student? How many books will be left over?



5. Last year, the Lakeville Bus System carried 85,425 passengers. If each bus carried an average of 15 passengers per trip, how many trips did the buses make?
6. A newspaper press prints the same number of papers each hour. If it prints 24,216 papers in 6 hours, how many papers does it print in 1 hour?

16 THE DOZENAL SYSTEM

Jim had some toothpicks that he wanted to arrange in *dozens*, or sets of 12. How many sets of 12 toothpicks could he make? How many toothpicks would remain?



Can you use the division algorithm to find the number of sets of 12 toothpicks Jim could make and the number of toothpicks remaining? Complete this equation.

$$25 = \underline{\quad} \times 12 + \underline{\quad}$$

Another name for the number 12 is dozen.

Another name for the number 24 is dozen.

Another name for the number 25 is dozen .

Express each number in dozens and ones.

1. $46 = \underline{\quad}$ dozen

2. $47 = \underline{\quad}$ dozen

3. $48 = \underline{\quad}$ dozen

4. $49 = \underline{\quad}$ dozen

5. $61 = \underline{\quad}$ dozen

6. $62 = \underline{\quad}$ dozen

7. $89 = \underline{\quad}$

8. $96 = \underline{\quad}$

9. $100 = \underline{\quad}$

10. $127 = \underline{\quad}$

11. $144 = \underline{\quad}$

12. $178 = \underline{\quad}$

13. $213 = \underline{\quad}$

14. $546 = \underline{\quad}$

15. $427 = \underline{\quad}$

16. $389 = \underline{\quad}$

17. $571 = \underline{\quad}$

18. $872 = \underline{\quad}$

Jim and Sue were asked to express the number 154 in dozens. Sue worked the exercise this way.

12	154	10
	120	
	34	
	24	2
	10	12

She said that $154 = 12$ dozen 10. Jim said he thought 154 should be named another way. Complete Jim's sentence.

$$\begin{aligned}154 &= 12 \text{ dozen } 10 \\&= 1 \underline{\quad} \text{ dozen } 10\end{aligned}$$

Express each number in two different ways.

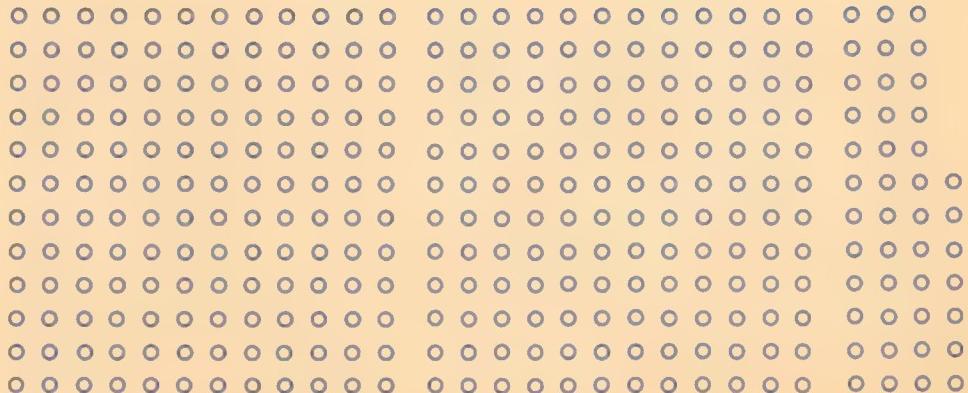
1. $144 = \underline{\quad}$ dozen
 $144 = \underline{\quad}$ dozen dozen

2. $153 = \underline{\quad}$ dozen $\underline{\quad}$
 $153 = \underline{\quad}$ dozen dozen $\underline{\quad}$

3. $178 = \underline{\quad}$ dozen $\underline{\quad}$
 $178 = \underline{\quad}$ dozen dozen $\underline{\quad}$ dozen $\underline{\quad}$

4. $245 = \underline{\quad}$ dozen $\underline{\quad}$
 $245 = \underline{\quad}$ dozen dozen $\underline{\quad}$ dozen $\underline{\quad}$

-
5. Study the diagram below.



$$\begin{aligned}331 &= \underline{\quad} \text{ dozen } \underline{\quad} \\&= \underline{\quad} \text{ dozen dozen } \underline{\quad} \text{ dozen } \underline{\quad}\end{aligned}$$

Using dots, draw an illustration for 1 dozen dozen 9 dozen 10. Draw a box around each dozen dozen and a ring around each dozen.

Jim said that another name for 1 dozen dozens is 1 *gross*. He expressed 291 in this way.

$$\begin{aligned} 291 &= 24 \text{ dozen } 3 \\ &= 2 \text{ dozen dozens } 3 \\ &= 2 \text{ gross } 3 \end{aligned}$$

12	291	20
	240	
	51	
	48	
3	24	

$$291 = \underline{\quad} \text{ dozen } \underline{\quad}$$

12	24	2
	24	
	0	

$$24 \text{ dozen } = \underline{\quad} \text{ gross } \underline{\quad}$$

$$\text{So, } 291 = \underline{\quad} \text{ gross } \underline{\quad}.$$

Express these numbers in the dozenal system.

1. $590 = \underline{\quad} \text{ gross } \underline{\quad} \text{ dozen } \underline{\quad}$

2. $642 = \underline{\quad} \text{ gross } \underline{\quad} \text{ dozen } \underline{\quad}$

3. $581 = \underline{\quad} \text{ gross } \underline{\quad} \text{ dozen } \underline{\quad}$

4. $880 = \underline{\quad} \text{ gross } \underline{\quad} \text{ dozen } \underline{\quad}$

5. $1283 = \underline{\quad}$

6. $1019 = \underline{\quad}$

7. $1235 = \underline{\quad}$

8. $1248 = \underline{\quad}$

9. $1460 = \underline{\quad}$

10. $1592 = \underline{\quad}$

11. $1765 = \underline{\quad}$

12. $1824 = \underline{\quad}$

13. $2024 = \underline{\quad}$

Express these numbers as standard numerals.

14. $6 \text{ dozen } 3 = \underline{\quad}$

15. $11 \text{ dozen } 8 = \underline{\quad}$

16. $1 \text{ gross } 7 = \underline{\quad}$

17. $3 \text{ gross } 7 \text{ dozen } = \underline{\quad}$

18. $3 \text{ dozen dozen } = \underline{\quad}$

19. $10 \text{ gross } = \underline{\quad}$

20. $5 \text{ gross } 7 \text{ dozen } 9 = \underline{\quad}$

21. $6 \text{ dozen gross } = \underline{\quad}$

Write the number that is 1 greater than each number below.

1. 3 dozen 4

2. 3 dozen 11

3. 11 dozen 11

4. 6 dozen 11

5. 1 gross 3 dozen 5

6. 5 gross 7 dozen 11

7. 8 gross 11 dozen 11

8. 11 gross 10 dozen 11

9. 11 gross 11 dozen 11

10. 16 gross

1 dozen gross = 1 great gross

Express each number in the dozenal system.

11. $1730 = ?$

12. $1897 = ?$

13. $2745 = ?$

14. $3772 = ?$

15. $4296 = ?$

16. $5227 = ?$

Express each number as a standard numeral.

17. 6 great gross 5 gross 1 dozen

18. 6 great gross 1 dozen dozen

19. 1 dozen dozen dozen

20. 2 great gross 5 dozen 7

21. 3 great gross 5 gross 7 dozen 8



Express each number in the dozenal system.

22. $2000 = ?$

23. $3248 = ?$

24. $1976 = ?$

25. $5280 = ?$

26. $3689 = ?$

27. $4949 = ?$

28. $7999 = ?$

29. $8420 = ?$

30. $7295 = ?$

Answer each question.

1. Sam has 3672 ants in his ant farm.
 - a. How many dozen does he have?
 - b. How many gross does he have?
 - c. How many great gross does he have?

- 2 Tess has 6783 kernels of popcorn.
 - a. How many dozen does she have?
 - b. How many gross does she have?
 - c. How many great gross does she have?

3. Seth has 5372 freckles.
 - a. How many gross does he have?
 - b. How many great gross does he have?

4. Lisa has 10,000 hairs on her head.
 - a. How many great gross does she have?
 - b. How many gross does she have?

5. Fred has 12,765 blades of grass on his front lawn.
 - a. How many gross does he have?
 - b. How many great gross does he have?

6. Al's house has 15,760 nails in it.
 - a. How many gross are there?
 - b. How many great gross are there?



Express each number in the dozenal system.

1. $129 = \underline{?}$

2. $1042 = \underline{?}$

3. $847 = \underline{?}$

4. $1440 = \underline{?}$

5. $11,999 = \underline{?}$

6. $98 = \underline{?}$

7. $924 = \underline{?}$

8. $7859 = \underline{?}$

Express each number in standard numerals.

9. 3 dozen 9 = ?

10. 2 gross 1 dozen = ?

11. 1 great gross 8 dozen 5 = ?

12. 3 great gross 4 = ?

13. 2 great gross 5 gross 2 dozen 11 = ?

14. 5 gross 3 dozen = ?

Answer each question. Express your answer in the dozenal system.

15. An orange grove produced 21,849 oranges in one harvest.
How many oranges were produced?

16. Fred estimated that the oak tree in his yard had 31,285 leaves.
How many leaves were there on the oak tree?

Copy and complete each chart.

1.	\times	8	9	12	10
7	?	?	?	?	?
5	?	?	?	?	?
9	?	?	?	?	?
20	?	?	?	?	?

2.	\times	10	11	8	30
13	?	?	?	?	?
24	?	?	?	?	?
20	?	?	?	?	?
100	?	?	?	?	?

Compute mentally.

3. $20 \times 60 = \underline{\quad}$

4. $8 \times 5000 = \underline{\quad}$

5. $50 \times 3000 = \underline{\quad}$

6. $7 \times 4000 = \underline{\quad}$

7. $700 \times 700 = \underline{\quad}$

8. $2000 \times 90 = \underline{\quad}$

9. $500 \times 80 = \underline{\quad}$

10. $50 \times 500 = \underline{\quad}$

11. $5000 \times 50 = \underline{\quad}$

12. $900 \times 900 = \underline{\quad}$

13. $20 \times 80 = \underline{\quad}$

14. $900 \times 200 = \underline{\quad}$

15. $30 \times 70 = \underline{\quad}$

16. $700 \times 30 = \underline{\quad}$

17. $4000 \times 80 = \underline{\quad}$

18. $60 \times 700 = \underline{\quad}$

19. $900 \times 500 = \underline{\quad}$

20. $70 \times 600 = \underline{\quad}$

Compute each quotient. Check your answers.

21. $18 \overline{)3724}$

22. $29 \overline{)5147}$

23. $37 \overline{)6208}$

24. $21 \overline{)3560}$

25. $17 \overline{)6284}$

26. $13 \overline{)4175}$

27. $29 \overline{)6824}$

28. $16 \overline{)4780}$

29. $31 \overline{)5575}$

30. $29 \overline{)3146}$

31. $37 \overline{)5384}$

32. $42 \overline{)3860}$

33. $41 \overline{)6794}$

34. $32 \overline{)3749}$

35. $56 \overline{)4327}$

